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COMMON FIXED POINT THEOREMS FOR INTERPOLATIVE RATIONAL-TYPE MAPPING IN COMPLEX-VALUED METRIC SPACE

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Abstract. This paper aims to demonstrate the common fixed point theorem for interpolative rational-type contraction mapping in complex-valued metric spaces. Also provide an example for verification of the proven results. Further, as an application, the paper proves the existence and uniqueness solution of the $R-L-C$ differential equation.

1. Introduction

In 1906, Fréchet introduced the concept of metric spaces. Metric space is used as a bridge among abstract spaces in mathematics. Banach [8] in his PhD thesis linked the fixed point theory and metric spaces which deals with various mathematical models, including biology, chemistry, computer science, economics, engineering, global analysis statistics, In 1975, Dass and Gupta [10] initiated the results on rational expressions in metric spaces and extended the Banach contraction principle. In 2011, Azam *et al.* [7] gave the concepts of new spaces called complex valued metric spaces and established the existence of fixed point theorems under the contraction condition in rational expression. The complex-valued metric space has several applications in the branches of Mathematics, including algebraic geometry, number theory applied Mathematics, hydrodynamics, mechanical engineering, thermodynamics and electrical engineering.

Recently, several authors have studied the existence and uniqueness of the fixed point and common fixed point theorems for self-mappings in complex-valued metric spaces. Pandey and Tiwari [33] proved a common fixed point theorem in complex-valued metric spaces. Analouei Adegani and Motamednezad [4] proved some common fixed point theorems in complex valued metric spaces. Marzouki *et al.* [29] gave a generalized common fixed point theorem in complex-valued b-metric spaces. Berrah *et al.* [9] proved common fixed point theorems of Meir-Keeler contraction type in complex valued metric space and an application to dynamic programming. Mani *et al.* [28] proved a common fixed point theorems on tri-complex valued metric space. Raj *et al.* [37] proved common fixed point theorems under rational contractions in complex-valued extended b-metric spaces. One can see in [2,25,26,29,32,38,42,43,45] and the references therein.

Another generalization of the Banach contraction principle was proposed by Karapinar [19] converted the classical Kannan [16] contraction to interpolative Kannan mapping in metric

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spaces. Since then, several finding has been done for various type of interpolative mapping in different spaces. Karapinar *et al.* [20] generalized the results on interpolative Hardy-Rogers type contractions. Yeşilkaya [48] gave the results on interpolative Hardy-Rogers contractive of Suzuki-type mappings. Gautam *et al.* [12] proved fixed point results for ω-interpolative Chatterjea type contraction in quasi-partial b-metric space. Mishra *et al.* [30] proved an interpolative Reich–Rus–Ćirić and Hardy–Rogers contraction on quasi-partial b-metric space and related fixed point results. Alansari and Ali [1] gave the results on interpolative prešić type contractions and related results. Wangwe and Kumar [47] proved fixed point results for interpolative ψ-Hardy-Rogers type contraction mappings in quasi-partial b-metric space with applications. For literature, we refer our readers in [3, 6, 11, 13, 16–19, 21–24, 31, 34, 35] and the references contained.

The new notion in this paper is to combine the concept of interpolative mapping by [19] with the complex-valued metric space due to [7]. To sustain our results we give an example. An application for an R-L-C differential equation is provided. The results obtained will be able to generalize several works from the literature such as [4, 31, 41–44] and several other.

2. Preliminaries

This section provide some preliminaries of definitions and theorems for enhancing the main results.

Azam *et al.* [7] gave the following definition on complex-valued metric spaces.

Definition 2.1. *Let $\mathbb{C}$ be a set of complex number and $z_1, z_2 \in \mathbb{C}$ define a partial order $\preceq$ on $\mathbb{C}$ as follows: $z_1 \preceq z_2$ if and only if one of the following conditions is satisfied:*

(C1) $\Re(z_1) = \Re(z_2), \Im(z_1) < \Im(z_2)$,
(C2) $\Re(z_1) < \Re(z_2), \Im(z_1) = \Im(z_2)$,
(C3) $\Re(z_1) < \Re(z_2), \Im(z_1) < \Im(z_2)$,
(C4) $\Re(z_1) = \Re(z_2), \Im(z_1) = \Im(z_2)$.

In particular, we will write $z_1 \precneqq z_2$ if $z_1 \neq z_2$ and all of $(C1), (C3)$ and $(C4)$ is satisfied and we will write $z_1 \preceq z_2$ if only $(C4)$ is satisfied.

Also, it is known that

(i) If $0 \preceq z_1 \precneqq z_2$, then $|z_1| < |z_2|$.
(ii) If $z_1 \preceq z_2$ and $z_2 \preceq z_3$, then $z_1 \prec z_3$.

The metric function in complex-valued metric space is as follows:

Definition 2.2. [7] Let X be a non-empty set. Assume that a mapping $d : X \times X \to \mathbb{C}$ is a metric on a complex-valued metric space if the following axioms hold:

(CM1) $0 \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0 \Longleftrightarrow x = y$,
(CM2) $d(x, y) = d(y, x)$ for all $x, y \in X$,
(CM3) $d(x, y) \preceq d(x, t) + d(t, y)$ for all $x, y, t \in X$.

Then d is called a complex-valued metric on X and (X, d) is called a complex-valued metric space.

The following are examples which satisfy the axioms of complex-valued metric space.

Example 2.1. [41] Let $X = \mathbb{C}$ be a se of complex number, define $d : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$. By

$$d(z_1, z_2) = |x_1 - x_2| + i|y_1 - y_2|,$$

where $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. Then $(\mathbb{C}, d)$ is a complex-valued metric space.

Example 2.2. [43] Let $X = \mathbb{C}$. Define a metric $d : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ by

$$d(z_1, z_2) = |z_1 - z_2|e^{ik},$$

where $k \in [0, \frac{\pi}{2}]$. Then $(\mathbb{C}, d)$ is a complex-valued metric space.

The following are topological convergence properties of the Cauchy sequence and completeness on complex-valued metric spaces.

Definition 2.3. [7] Suppose that (X, d) is a complex-valued metric space.

(i) A sequence $\{x_n\}$ converges to an element $x \in X$ if for every $0 \prec t \in \mathbb{C}$ there exist an integer $\mathbb{N}$ such that

$$d(x_n, x) \prec t,$$

for all $n \geq \mathbb{N}$. We write this by

$$\lim_{n\to\infty} d(x_n, x) \implies x_n \to x \ as \ n \to \infty.$$

(ii) If for any $t \in \mathbb{C}$ with $0 \prec t$, there exist $N \in \mathbb{N}$ such that, for all $n > N$,

$$d(x_n, x_{n+m}) \prec t,$$

where $m \in \mathbb{N}$, then $\{x_n\}$ is called a Cauchy sequence in X.

(iii) If every Cauchy sequence in X is convergent, then (X, d) is said to be a complete complex-valued metric space.

Lemma 2.1. [7] Let (X, d) be a complex-valued metric space and (x_n) be a sequence in X. Then $\{x_n\}$ converges to a point $x \in X$ if and only if

$$|d(x_n, x)| \to 0 \ as \ n \ \to \infty.$$

Lemma 2.2. [7] Let (X, d) be a complex-valued metric space and (x_n) be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if

$$|d(x_n, x_{n+m})| \to 0 \ as \ n \ \to \infty.$$

Definition 2.4. *[15, 40] Let Υ and Γ be two self-mapping of a non-empty set X.*

(i) *A point $\varpi \in X$ is called a fixed point of Υ if $\Upsilon\varpi = \varpi$.*

(ii) *A point $\varpi \in X$ is called a coincidence point of Υ and Γ if $\Upsilon\varpi = \Gamma\varpi$ and the point $\varpi \in X$ such that $\varpi = \Upsilon\varpi = \Gamma\varpi$ is called point of coincidence of Υ and Γ.*

(iii) *A point $\varpi \in X$ is called a common fixed point of Υ and Γ if $\varpi = \Upsilon\varpi = \Gamma\varpi$.*

The following are some preliminary results:
Pitchaimani and Saravanan [36] gave the following definition and theorem.

Definition 2.5. *[36] In a complete b-metric space (Ω, d, s), a mapping $T : \Omega \to \Omega$ is called an interpolative Ćirić-Reich-Rus-type contraction, if there are constants $\lambda \in [0, 1)$ and $\delta, \vartheta \in (0, 1)$ such that*

$$\frac{1}{s}d(\Upsilon x, \Upsilon y) \;\leq\; \lambda[d(x, y)]^{\delta}.[d(x, \Upsilon x)]^{\vartheta}.[d(y, \Upsilon y)]^{1-\delta-\vartheta}, \tag{1}$$

for all $x, y \notin Fix_{\Upsilon}(X)$, where $Fix_{\Upsilon}(X)$ denotes the set of all fixed points of Υ.

Theorem 2.1. *[36] Suppose a self mapping $\Upsilon : X \to X$ is an interpolative Ćirić-Reich-Rus-type contraction on a complete b-metric space (X, d, s). Then T has a fixed point in X.*

Azam *et al.* [7] proved the following results on complex-valued metric space:

Theorem 2.2. *[7] Let (X, d) be a complete complex-value metric space and $\Upsilon, \Gamma : X \to X$ be two mappings. If Υ and Γ satisfy*

$$d(\Upsilon x, \Gamma y) \;\preceq\; \lambda d(x, y) + \mu\frac{d(x, \Upsilon x)d(y, \Gamma y)}{1 + d(x, y)},$$

for all $x, y \in X$, where λ, μ are non-negative real with $\lambda + \mu < 1$, then Υ and Γ have a unique common fixed point in X.

Definition 2.6. *[31] Suppose (X, d) be a metric space. Furthermore, consider a continuous operator $\Upsilon : X \to X$. If exists $\lambda, \alpha(0, 1)$, such that*

$$d(\Upsilon x, \Upsilon y) \;\leq\; \lambda\left[\frac{d(x, \Upsilon x)d(y, \Upsilon y)}{d(x, y)}\right]^{\alpha}[d(x, y)]^{1-\alpha},$$

for distinct $x, y \in X/Fix_{\Upsilon}(X)$. Then Υ is known as interpolative Dass and Gupta rational type contraction, where $Fix_{\Upsilon}(X)$ denotes the set of all fixed points of Υ.

Theorem 2.3. *[31] Let (X, d) be a complete metric space and Υ be an interpolative rational-type contraction. Then Υ possesses a fixed point.*

3. Main Results

This section gives the main results:

Theorem 3.1. *Let (X, d) be a complete complex-value metric space and $\Upsilon, \Gamma : X \to X$ be the two interpolative rational-type contraction mappings. If the following inequality holds:*

$$d(\Upsilon x, \Gamma y) \;\preceq\; \vartheta\left[\frac{d(x, \Upsilon x)d(y, \Gamma y)}{d(x, y)}\right]^{\delta}[d(x, y)]^{1-\delta}, \tag{2}$$

for all $x, y \in X$ and $\delta \in (0, 1)$. Then, Υ and Γ possess a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X by the following way:

$$x_{2n+1} \;=\; \Upsilon x_{2n},$$

and

$$x_{2n+2} \;=\; \Gamma x_{2n+1},$$

for all $n \geq 0$. If $x_{2n+1} = \Upsilon x_{2n} = \Gamma x_{2n+1}$, our proof is completed and $x_{2n+1} = x_{2n}$. Thus, x_{2n} is a unique common fixed point of Υ and Γ. On contrary to that, suppose $x_{2n+1} \neq \Upsilon x_{2n}$ and $x_{2n+2} \neq \Gamma x_{2n+1}$. Which is equivalent to

$$d(x_{2n+1}, x_{2n+2}) = d(\Upsilon x_{2n}, \Gamma x_{2n+1}) \neq 0, \tag{3}$$

for all $n \geq 0$.

Let $x = x_{2n}$ and $y = x_{2n+1}$ in (2), we have

$$\begin{aligned}
d(\Upsilon x_{2n}, \Gamma x_{2n+1}) &\preceq \vartheta \left[\frac{d(x_{2n}, \Upsilon x_{2n}) d(x_{2n+1}, \Gamma x_{2n+1})}{d(x_{2n}, x_{2n+1})} \right]^{\delta} [d(x_{2n}, x_{2n+1})]^{1-\delta}, \\
d(x_{2n+1}, x_{2n+2}) &\preceq \vartheta \left[\frac{d(x_{2n}, x_{2n+1}) d(x_{2n+1}, x_{2n+2})}{d(x_{2n}, x_{2n+1})} \right]^{\delta} [d(x_{2n}, x_{2n+1})]^{1-\delta}, \\
d(x_{2n+1}, x_{2n+2}) &\preceq \vartheta \Big[d(x_{2n}, x_{2n+2}) \Big]^{\delta} [d(x_{2n}, x_{2n+1})]^{1-\delta}, \\
\Big[d(x_{2n+1}, x_{2n+2}) \Big]^{1-\delta} &\preceq \vartheta \Big[d(x_{2n}, x_{2n+1}) \Big]^{1-\delta}, \\
|d(x_{2n+1}, x_{2n+2})| &\preceq \vartheta^{\frac{1}{1-\delta}} |d(x_{2n}, x_{2n+1})|. \qquad (4)
\end{aligned}$$

Similarly,

$$d(x_{2n+2}, x_{2n+3}) = d(\Upsilon x_{2n+1}, \Gamma x_{2n+2}) \neq 0.$$

Let $x = x_{2n+1}$ and $y = x_{2n+2}$ in (2), we get

$$|d(x_{2n+2}, x_{2n+3})| \preceq \vartheta^{\frac{1}{1-\delta}} |d(x_{2n+1}, x_{2n+2})|. \tag{5}$$

From (4) in (5), we obtain

$$|d(x_{2n+2}, x_{2n+3})| \preceq \vartheta^{\frac{2}{1-\delta}} |d(x_{2n}, x_{2n+1})|, \tag{6}$$

for all $n \in \mathbb{N}$.

By repeating the above steps through mathematical induction, for all $n \geq 0$ we have

$$|d(x_{2n+1}, x_{2n+2})| \preceq \vartheta^{\frac{n}{1-\delta}} |d(x_{2n}, x_{2n+1})|. \tag{7}$$

Therefore,

$$\lim_{n\to\infty} |d(x_{2n+1}, x_{2n+2})| = 0, \tag{8}$$

which is a contradiction.

By Lemma 2.1 and Definition 2.3

$$\lim_{n\to\infty} |d(x_{2n+1}, x_{2n+2})| \to 0 \ as \ n \to \infty.$$

Next, we show that $\{x_{2n}\}$ is a Cauchy sequence. Since $|d(x_{2n+1}, x_{2n+2})| = 0$. Using Definition 2.3, Lemma 2.2, $(CM3)$ and inequality (7) for any positive integer m and n with $m > n$, we have

$$\begin{aligned}
d(x_{2n+1}, x_{2n+m}) &\preceq d(x_{2n+1}, x_{2n+2}) + d(x_{2n+2}, x_{2n+m}),\\
&\preceq d(x_{2n+1}, x_{2n+2}) + d(x_{2n+2}, x_{2n+3}) + \cdots + d(x_{n+m-1}, x_{n+m}),\\
&\preceq \vartheta^{\frac{n}{1-\delta}}|d(x_{2n+1}, x_{2n+2})| + \vartheta^{\frac{n+1}{1-\delta}}|d(x_{2n+1}, x_{2n+2})| + \cdots +\\
&\quad \vartheta^{\frac{n+m-1}{1-\delta}}|d(x_{2n+m-1}, x_{2n+m})|,\\
&\preceq [\vartheta^{\frac{n}{1-\delta}} + \vartheta^{\frac{n+1}{1-\delta}} + \cdots + \vartheta^{\frac{n+m-1}{1-\delta}}]|d(x_{2n+1}, x_{2n+2})|,\\
&\preceq \vartheta^{\frac{n}{1-\delta}}[1 + \vartheta^{\frac{1}{1-\delta}} + \cdots + \vartheta^{\frac{n+m-1}{1-\delta}}]|d(x_{2n+1}, x_{2n+2})|,\\
&\preceq \frac{\vartheta^{\frac{n}{1-\delta}}}{1-\vartheta^{\frac{1}{1-\delta}}}|d(x_{2n+1}, x_{2n+2})|. \qquad (9)
\end{aligned}$$

By taking $n \to \infty$ in (10), using Lemma 2.2 we get

$$|d(x_{2n+1}, x_{2n+m})| \to 0,$$

which deduce that the sequence $\{x_{2n}\}$ for all $n \in \mathbb{N}$ is a Cauchy sequence.

Since (X, d) is a complete complex valued-metric space, there exists a fixed point $\varpi \in X$ such that

$$\lim_{n\to\infty} d(x_{2n}, \varpi) = 0.$$

Now, we claim that $\Upsilon\varpi \neq \varpi$. By the concept of complex-valued metric d, using $(CM3)$ and Definition 2.4, we get

$$\begin{aligned}
d(\varpi, \Upsilon\varpi) &\preceq d(\varpi, x_{2n+2}) + d(x_{2n+2}, \Upsilon\varpi),\\
&\preceq d(\varpi, x_{2n+2}) + d(\Gamma x_{2n+1}, \Upsilon\varpi),\\
&\preceq d(\varpi, x_{2n+2}) + d(\Upsilon\varpi, \Gamma x_{2n+1}). \qquad (10)
\end{aligned}$$

Let $x = \varpi$ and $y = x_{2n+1}$ in (2), then inequality (10) deduce to

$$\begin{aligned}
d(\varpi, \Upsilon\varpi) &\preceq d(\varpi, x_{2n+2}) + \vartheta\left[\frac{d(\varpi, \Upsilon\varpi)d(x_{2n+1}, \Gamma x_{2n+1})}{d(\varpi, x_{2n+1})}\right]^{\delta}[d(\varpi, x_{2n+1})]^{1-\delta},\\
&\preceq d(\varpi, x_{2n+2}) + \vartheta\left[\frac{d(\varpi, \Upsilon\varpi)d(x_{2n+1}, \Gamma x_{2n+1})}{d(\varpi, x_{2n+1})}\right]^{\delta}[d(\varpi, x_{2n+1})]^{1-\delta},\\
&\preceq d(\varpi, x_{2n+2}) + \vartheta\left[\frac{d(\varpi, \Upsilon\varpi)d(x_{2n+1}, x_{2n+2})}{d(\varpi, x_{2n+1})}\right]^{\delta}[d(\varpi, x_{2n+1})]^{1-\delta},\\
&\preceq d(\varpi, \varpi) + \vartheta\left[\frac{d(\varpi, \Upsilon\varpi)d(\varpi, \varpi)}{d(\varpi, \varpi)}\right]^{\delta}[d(\varpi, \varpi)]^{1-\delta},\\
|d(\varpi, \Upsilon\varpi)| &= 0,
\end{aligned}$$

which is a contradiction. This implies that $d(\varpi, \Upsilon\varpi) = 0$. Thus $\varpi = \Upsilon\varpi$. Similarly, we can show that $\varpi = \Gamma\varpi$. Hence ϖ is a common fixed point of Υ and Γ.

For uniqueness of ϖ in Υ and Γ. Assume that there exist a distinct common fixed point $\varpi^\star \neq \varpi$ such that $\varpi^\star = \Upsilon\varpi^\star = \Gamma\varpi^\star$. Let $x = \varpi$ and $y = \varpi^\star$ in (2), then inequality (10) deduce to

$$\begin{aligned} d(\varpi,\varpi^\star) &= d(\Upsilon\varpi,\Gamma\varpi^\star),\\ d(\varpi,\varpi^\star) &\preceq \vartheta\left[\frac{d(\varpi,\Upsilon\varpi)d(d(\varpi,\varpi^\star),\Gamma d(\varpi,\varpi^\star))}{d(\varpi,d(\varpi,\varpi^\star))}\right]^\delta [d(\varpi,d(\varpi,\varpi^\star))]^{1-\delta},\\ |d(\varpi,\varpi^\star)| &= 0. \end{aligned}$$

Therefore, we have $\varpi=\varpi^\star$ and thus ϖ is a unique common fixed point of Υ and Γ. This proof is completed. □

For the creativity of Theorem 3.1, we provide the corollary below.

Corollary 3.1. Let (X,d) be a complex-valued metric space and $x_0\in X$. Let $\Upsilon,\Gamma:X\to X$ be an interpolative rational-type contraction mapping, then the following condition holds:

$$d(\Upsilon x,\Gamma y) \;\leq\; \vartheta_1\Big[d(x,y)\Big]^\sigma.\vartheta_2\left[\frac{d(x,\Upsilon x)d(y,\Gamma y)}{d(x,y)}\right]^\delta.\vartheta_3\left[\frac{d(y,\Upsilon y)d(x,\Gamma x)}{d(x,y)}\right]^{1-\sigma-\delta},$$

for all $x,y\in X$, where $\sum_1^3\vartheta_i<1$ and $0<\delta+\vartheta\leq 1$.

Then, there exists a unique common fixed point of Υ and Γ in X.

Proof. The proof of this corollary follows the similar proof of Theorem 3.1. This completes the proof. □

Corollary 3.2. Let (X,d) be a complex-valued metric space. Let $\Upsilon,\Gamma:X\to X$ be an interpolative Ćirić-Reich-Rus-type contraction mapping such that

$$d(\Upsilon x,\Gamma y) \;\leq\; \eta[d(x,y)]^\delta.[d(x,\Gamma x)]^\vartheta.[d(y,\Upsilon y)]^{1-\delta-\vartheta},$$

for all $x,y\in X\backslash Fix(\Upsilon,\Gamma)$, where $\eta,\delta,\vartheta\in(0,1)$, and $0<\eta+\delta+\vartheta\leq 1$.

Then Υ and Γ admit a unique common fixed point in X.

Proof. The proof of this corollary follows the similar proof of Theorem 3.1. This completes the proof. Hence Υ and Γ admit a unique common fixed point in X. □

The following is an example for verification of the results proved above.

Example 3.1. *Let $X=[0,1)$ and $d:X\times X\to\mathbb{C}$ be defined by*

$$d(x,y) = |x-y|e^{ik},$$

where $k\in[0,\frac{\pi}{2}]$. Then, (X,d) is a complete complex-valued metric space.

Let $\Upsilon,\Gamma:X\to X$ be pair of self-mapping defined by

$$\Upsilon x = x^2.$$

and

$$\Gamma x = \frac{x}{4},$$

for all $x, y \in X$. Note that $e^{ik} = \cos k + i \sin k$. To validate the hypothesis imposed in Theorem 3.1, we start by calculating the following complex-valued metric.

$$
\begin{aligned}
d(\Upsilon x, \Gamma y) &= d\Big(x^2, \frac{y}{4}\Big) = \Big|x^2 - \frac{y}{4}\Big| e^{i\frac{\pi}{2}} = \Big|\frac{4x^2 - y}{4}\Big| e^{i\frac{\pi}{2}} \\
&\Big|\frac{4x^2 - y}{4}\Big|\Big(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\Big) = \Big|\frac{4x^2 - y}{4}\Big| i, \qquad (11)\\
d(x, y) &= |x - y| e^{i\frac{\pi}{2}} = |x - y|\Big(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\Big) = |x - y| i, \qquad (12)\\
d(x, \Upsilon x) &= |x - x^2| e^{i\frac{\pi}{2}} = |x - x^2|\Big(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\Big) = |x - x^2| i, \qquad (13)\\
d(y, \Gamma y) &= d\Big(y, \frac{y}{4}\Big) = \Big|y - \frac{y}{4}\Big| e^{i\frac{\pi}{2}} = \Big|\frac{4y - y}{4}\Big| e^{i\frac{\pi}{2}} \\
&= \Big|\frac{3y}{4}\Big|\Big(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\Big) = \Big|\frac{3y}{4}\Big| i. \qquad (14)
\end{aligned}
$$

Using (11) − (14) and $\delta = 0.25, x = 0.5, y = \frac{2}{3}, \vartheta = 0.5$ in (2), we get

$$
\begin{aligned}
\Big|\frac{4x^2 - y}{4}\Big| i &\le \vartheta\left[\frac{|x - x^2| i |0.75y| i}{|x - y| i}\right]^{\delta}\Big[|x - y| i\Big]^{1-\delta}, \\
\Big|\frac{1}{12}\Big| i &\le \frac{1}{2}\left[\frac{|0.25| i |0.5| i}{|0.16666| i}\right]^{0.25}\Big[|0.16666| i\Big]^{0.75}, \\
0.083333i &\le \frac{1}{2}\left[\frac{|0.25| i |0.5| i}{|0.16666| i}\right]^{0.25}\Big[|0.16666| i\Big]^{0.75}, \\
0.083333i &\le \frac{1}{2}\Big[0.75i\Big]^{0.25}\Big[|0.16666| i\Big]^{0.75}, \\
0.083333i &\le 0.121369301i, \\
0.083333 &\le 0.121369301.
\end{aligned}
$$

Hence, the hypothesis of Theorem 3.1 is verified. Thus, a pair of mappings Υ and Γ has unique common fixed points in X. Therefore 0 is a unique common fixed point of Υ and Γ.

4. An Application of R-L-C-Circuit in Complex-valued Metric Spaces

In this section, aims to investigate the relation between $R - L - C$ series circuit in finding a common solution between inductance and capacitance during the flow of current in the circuit where we can apply Theorem 3.1. The proof involves the second-order differential equation by transforming it into the system of integral equations. Electrical engineers benefit from complex numbers when they deal with the fact that the current through a circuit element such as a capacitor or inductor is not in phase with the voltage across it, specifically in the solution of electric equations. This type of work is motivated by [5, 14, 27, 39, 46]. The electric circuit can be represented by ternary relation, R a resistor has a unit Ohms (Ω), L an inductor has a unit Henry (H) and C a capacitor has a unit Farad (F) on V source of power in a series circuit measured in Volt (V).

If the rate of charge q in condenser concerning time t is denoted by current I, that is amount to say $I = \frac{dq}{dt}$. We get the following ternary relations:

$$\begin{aligned} V_R &= IR, \\ V_C &= \frac{q}{C}, \\ V_L &= L\frac{dI}{dt}. \end{aligned}$$

By Kirchhoff's Voltage law, the sum of the voltage across the loop is equal to the sum of every voltage drop in the same loop.

$$V_m = V_R + V_C + V_L.$$

The differential equation for $R-L-C$-circuit can be presented by:

$$\begin{aligned} IR + \frac{q}{C} + L\frac{dI}{dt} &= V_m(t), \\ L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} &= V_m(t), \end{aligned} \tag{15}$$

where $V_m(t)$ is the maximum voltage applied at time t. The characteristic equation of (15) is given by

$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0.$$

with the initial conditions assumed to be

$$q(0) = 0, \frac{dq(0)}{dt} = 0\ ,\ R^2 > \frac{4L}{C}\ and\ \tau = \frac{R}{2L}.$$

The Green function generated for the differential equation (15) is given by

$$G(t,s) = \begin{cases} -se^{\tau(s-t)}, & 0 \le s \le t \le 1, \\ \\ -te^{\tau(s-t)}, & 0 \le t \le s \le 1, \end{cases}$$

Consider the following systems of equations. The first integral equation represents Inductance (L) and the second integral equation represent capacitance C. Assume that $x = L, y = C$, we have

$$\begin{cases} x(t) = \int_0^T G(t,s)K_1(t,s,x(s))ds + h(t), \\ \\ y(t) = \int_0^T G(t,s)K_2(t,s,y(s))ds + h(t), \end{cases} \tag{16}$$

for all $t, s \in X = [0,T], T > 0$, where $K : \mathcal{X} \times \mathcal{X} \times \mathbb{R}^n \to \mathbb{C}$ and $h : X \to \mathbb{R}^n$. Let $X = C(X, \mathbb{R}^n)$ on $\mathbb{C}$, set of all continuous mappings from $X \to \mathbb{R}^n$.

Define a complex-valued metric on X, by $d : X \times X \to \mathbb{C}$ and

$$d(x,y) = \max_{t\in[0,T]} |x-y|e^{ik}, k \in [0, \frac{\pi}{2}].$$

Then (X, d) is a complete complex-value metric space.

Theorem 4.1. Assume that the hypothesis below holds:

(i) $K_1, K_2 : [0,T] \times \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ are continuous,

(ii) there exists some function $G : [0, T] \times [0, T] \times [0, T] \to [0, \mathbb{R}]$, for each $x, y \in X$, $\delta \in [0, 1]$ and $t, s \in [0, T]$ with $\vartheta \in [0, 1]$ such that

$$|K_1(t, s, x(s)) - K_2(t, s, y(s))| \leq G(t, s)\mathcal{M}_{\mathcal{C}}(x, y), \tag{17}$$

where $\max_{0 \in [0,T]} |x(t) - y(t)|e^{ik} = \mathcal{M}_{\mathcal{C}}(x, y) = d(x, y)$ and

$$\mathcal{M}_{\mathcal{C}}(x, y) = \vartheta\left[\frac{d(x, \Upsilon x)d(y, \Gamma y)}{d(x, y)}\right]^{\delta}\Big[d(x, y)\Big]^{1-\delta}. \tag{18}$$

(iv) there exists $\vartheta \in [0, 1)$ such that

$$\frac{1 - e^{-\tau t} - \tau t e^{\tau(1-t)}}{\tau^2} \leq \vartheta, \tag{19}$$

where $\tau = \frac{R}{2L}$.

Then, the system of an electric differential equation 15 has a unique common solution, which is a solution of the integral equation 16.

Proof. We define self-mappings $\Upsilon, \Gamma : X \to X$ by

$$\begin{aligned} \Upsilon x(t) &= \int_0^T G(t, s)K_1(t, s, x(s))ds + h(t), \\ \Gamma y(t) &= \int_0^T G(t, s)K_2(t, s, y(s))ds + h(t), \end{aligned} \tag{20}$$

for all $t, s \in X$.

For $x, y \in C([0, T])$ with $x < y$, we assume that $d(\Upsilon x, \Gamma y) \geq \vartheta(\mathcal{M}_{\mathcal{C}}(x, y))$. Then, we show that Υ and Γ have a unique common fixed point in X. By the hypothesis $(i), (ii) and (iii)$, we have

$$\begin{aligned} |\Upsilon x(t) - \Gamma y(t)| &\leq \max_{0 \in [0,T]} \left| \left[\int_0^T G(t, s)K_1(t, s, x(s))ds + g(t)\right] - \right. \\ &\qquad \left. \left[\int_0^T G(t, s)K_2(t, s, y(s))ds + g(t)\right] \right|, \\ &\leq \max_{0 \in [0,T]} \left| \int_0^T G(t, s)(K_1(t, s, x(s)) - K_2(t, s, y(s)))ds \right|, \\ &\leq \max_{0 \in [0,T]} |\ K_1(t, s, x(s))x(t) - K_1(t, s, x(s))y(t)| \int_0^T G(t, s)ds \\ &\leq \max_{0 \in [0,T]} |x(t) - y(t)|e^{ik} \left[\int_0^t -se^{\tau(s-t)}ds - \int_0^1 te^{\tau(s-t)}ds\right], \\ &\leq \max_{0 \in [0,T]} |x(t) - y(t)|e^{ik} \left[\frac{1 - e^{-\tau t} - \tau t e^{\tau(1-t)}}{\tau^2}\right], \\ d(\Upsilon x, \Gamma y) &\leq \vartheta(\mathcal{M}_{\mathcal{C}}(x, y)), \end{aligned} \tag{21}$$

Consequently, we have

$$d(\Upsilon x, \Gamma y) \leq \vartheta\left[\frac{d(x, \Upsilon x)d(y, \Gamma y)}{d(x, y)}\right]^{\delta}\Big[d(x, y)\Big]^{1-\delta}, \tag{22}$$

which is a contradiction. Therefore $x = \Upsilon x = \Gamma y$ is a unique common fixed of Υ and Γ, also a solution to integral equation (16) and a second order differential equation (15). Hence, the hypothesis in Theorem 3.1 and Theorem 4.1 are satisfied. So, the proof is completed.

5. Conclusions

The main contribution of this study to fixed point theory is the fixed point result given in Theorem 3.1. This theoremcombine the concept of interpolative mapping by [19] with the complex-valued metric space due to [7]. The results obtained will be able to generalize several works from the literature such as [4, 31, 41–44] and several other. Also provide an illustrative example to support the results and an application to an R-L-C differential equation.

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