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FIXED POINT AND COMMON FIXED POINT THEOREMS FOR (γ, s, q)-F-CONTRACTION MAPPINGS IN b-METRIC LIKE SPACES WITH APPLICATION

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ABSTRACT. This paper proves some fixed point theorems for (γ, s, q)-F-Kannan mappings and a common fixed point theorem for (γ, s, q)-F-Reich type contraction mappings in a b-metric like space. Finally, we give an application to the solvability of a nonlinear integral equation.

1. INTRODUCTION

Since the introduction of the Banach contraction principle in 1922 [9], a source of knowledge in the field of abstract function spaces has remained. The Banach contraction principle is used to establish the existence and uniqueness of the solution to non-linear equations. Which is applied to solving real-world problems arising in mathematics, physics, biology and chemistry.

In 1968, Kannan [27] gave an extended concept of the Banach contraction principle as follows:

Theorem 1.1. [27] Let (Υ, d) be a complete metric space and a self mapping $\mu : \Upsilon \longrightarrow \Upsilon$ be a mapping such that

$$d(\mu\iota, \mu\kappa) \leq \lambda\{d(\iota, \mu\iota) + d(\kappa, \mu\kappa)\}, \tag{1}$$

for all $\iota, \kappa \in \Upsilon$ and $0 \leq \lambda \leq \frac{1}{2}$, then μ has a unique fixed point $z \in \Upsilon$ and for any $\iota \in \Upsilon$ the sequence of iterate $\{\mu^n\iota\}$ converges to z.

The following inequality is an equivalent form of (1):

$$d(\mu\iota, \mu\kappa) \leq \frac{\lambda}{2}\{d(\iota, \mu\iota) + d(\kappa, \mu\kappa)\}, \tag{2}$$

for some $\lambda \in [0, 1)$.

The next results were established by Reich [37].

Theorem 1.2. [37] Let Υ be a complete metric space with metric d, and let $\mu : \Upsilon \to \Upsilon$ be a function with the following property:

$$d(\mu\iota, \mu\kappa) \leq ad(\iota, \mu\iota) + bd(\kappa, \mu\kappa) + cd(\iota, \kappa), \tag{3}$$

for all $\iota, \kappa \in \Upsilon$ where a, b, c are nonnegative and satisfy $a + b + c < 1$. Then μ has a unique fixed point.

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Note that $a = b = 0$ yields Banach's fixed point theorem, while $a = b$ and $c = 0$ yield Kannan's fixed point theorem.

In 1993, Czerwik [18] established b-metric spaces by weakening the triangle inequality and generalized Banach's contraction principle to these spaces. Since then, several papers have been published on the fixed point theory of various classes of the single and multivalued maps in b-metric space. Alamgir *et al.* [4] proved a Mizoguchi-Takahashi type fixed point theorem in complete extended b-metric spaces. Chifu and Karapınar [14] gave a generalization of the Banach contraction principle in admissible hybrid Z-contractions in b-metric spaces. Chifu and Petruşel [15] gave the proof of some fixed point results for multi-valued Hardy-Rogers contractions in b-metric spaces. Kajanto and Lukacs [25] gave a note on the paper "Contraction mappings in b-metric spaces" by Czerwik. Kamran *et al.* [26] gave a generalization of b-metric space and some fixed point theorems. Kirk and Shahzad [29] gave a discussion on fixed point theory in b-distance spaces. Roshan *et al.* [38] proved a common fixed point theorems for weakly isotone increasing mappings in ordered b-metric spaces.

Likewise, Amini-Harandi [7] generalized the concept of partial metric space by introducing the metric-like space. Since then, several authors interested in working in metric-like spaces. Aydi and Karapinar [8] proved fixed point results for generalized α-ϕ-contractions in metric-like spaces and its applications. Hussain *et al.* [23] gave proof iof some fixed point results for contractive mappings in b-metric-like spaces. Khammahawong and Kumam [28] proved some fixed point theorems for generalized Hardy-Roger type F-contraction mappings in a metric-like space with an application to second-order differential equations.

Furthermore, Alghamdi *et al.* [5] introduced the notion of b-metric-like spaces by combining the concepts of b-metric space and metric-like space. Since then, several researchers have followed this new generalized metric space in many directions. Abbas *et al.* [2] proved common fixed points of (α-ψ)-generalized rational multivalued contractions in dislocated quasi b-metric spaces and presented some applications. Chen *et al.* [13] proved a common fixed point theorem concerning F-contraction in b-metric-like spaces. Cvetkovic *et al.* [17] gave some fixed point results on quasi-b-metric-like spaces. Joshi *et al.* [24] gave the existence results for integral equations and boundary value problems via fixed point theorems for generalized contractions in metric-like spaces. Mohammadi *et al.* [34] proved some results on fixed points for α-φ-Ciric generalized multi-function contractions. Rasham *et al.* [36] proved some multivalued fixed point results in dislocated b-metric spaces with application to a system of nonlinear integral equations. Zoto *et al.* [43] proved some common fixed point theorems for a class of (s, q)-contractive mappings in b-metric-like spaces with an application to integral equations. Sanatee *et al.* [39] proved some fixed point theorems in regular modular metric spaces and application to Caratheodory's type anti-periodic boundary value problem. Agarwal *et al.* [3] gave the study of fixed point theory and applications. Debnath *et al.* [19] gave the theory in fixed point theory and fractional calculus.

In 2012, Wardowski [41] gave an interesting generalization of Banach's fixed point theorem using a different type of contractions called F-contraction. Since then, many researchers have followed his approach to constructing new fixed-point theorems. Wardowski and Van Dung [42] gave a result of fixed points for F-weak contractions on complete metric spaces. Minak *et al.* [33] gave some results on ćirić type generalized F-contractions in complete metric spaces. Piri and

Kumam [35] proved some fixed point theorems concerning F-contraction in complete metric spaces. Altun *et al.* [6] proved a version of multivalued F-contractions in complete metric spaces. Cosentino *et al.* [16] gave a solvability of integrodifferential problems via fixed point theorems in b-metric spaces. Secelean [40] gave results on Weak F-contractions. Also, Durmaz *et al.* [20] proved some results in fixed points of ordered F-contractions. Goswami *et al.* [21] gave some results on F-contractive type mappings in b-metric spaces and some related fixed point results. Lukács and Kajántó [30] proved some fixed point theorems for various types of F-contractions in complete b-metric spaces. Mani *et al.* [31] gave an application and fixed point theorems for orthogonal generalized F-contraction mappings on O-complete metric space.

This paper aims to prove a generalized result for a fixed point theorem using (γ, s, q)-F-Kannan mappings due to Batra *et al.* [10] and a common fixed point theorem using (γ, s, q)-F-Reich type contraction mappings due to Hammad and De la Sen [22] and Joshi *et al.* [24] in b-metric-like spaces.

2. Preliminaries

We now introduce the preliminary definitions and theorems that will be useful in this paper. In 1993, Czerwik [18] gave a generalization of metric space to b-metric space as bellow:

Definition 2.1. [18] Let Υ be a non-empty set and $s \geq 1$ be a given real number. A function $d : \Upsilon \times \Upsilon \to [0, \infty)$ is called a b-metric if for all $\iota, \kappa, z \in \Upsilon$ the following conditions satisfied:

(B1) $d(\iota, \kappa) = 0$ if $\iota = \kappa$,
(B2) $d(\iota, \kappa) = d(\kappa, \iota)$ and
(B3) $d(\iota, \kappa) \leq s[d(\iota, z) + d(z, \kappa)]$.

The pair (Υ, d) is called a b-metric space. The number $s \geq 1$ is called the coefficient of (Υ, d).

Example 2.1. [11] Let $p \in (0, 1)$, and Let

$$\Upsilon = l_p(\mathbb{R}) := \Big\{ \iota = \{\iota_n\} \subset \mathbb{R} : \sum_{n=1}^{\infty} |\iota_n|^p \leq \infty \Big\},$$

together with the functional $d : l^p(\mathbb{R}) \times l^p(\mathbb{R}) \to \mathbb{R}$

$$d(\iota, \kappa) = \Big(\sum_{n=1}^{\infty} |\iota_n - \kappa_n|^p \Big)^{\frac{1}{p}},$$

where $\iota = \iota_n, \kappa = \kappa_n$. Then (Υ, d) is a b-metric space with $s = 2^{\frac{1}{p}}$.

In 2012, Amini-Harandi [7] gave a generalization of partial metric spaces to metric-like spaces by introducing the following properties.

Definition 2.2. [7] A metric-like space is a pair (Υ, σ) consisting of a non-empty set Υ together with a function $\sigma : \Upsilon \times \Upsilon \to \mathbb{R}^+$, such that for all $\iota, \kappa, z \in \Upsilon$ we have the following condition hold:

(σ1) $\sigma(\iota, \kappa) = 0 \Rightarrow \iota = \kappa$;
(σ2) $\sigma(\iota, \kappa) = \sigma(\iota, \kappa)$; and
(σ3) $\sigma(\iota, \kappa) \leq \sigma(\iota, \kappa) + \sigma(\kappa, z)$.

Then σ is called a metric-like space on Υ and the pair (Υ, σ) is called a metric-like space.

A metric-like on Υ satisfies all of the conditions of metric except that $\sigma(\iota,\iota)$ may be positive for $\iota \in \Upsilon$.

Remark 2.1. Every partial metric space is a metric-like space. This can be illustrated by the use of the following example:

Example 2.2. [7] Let $\Upsilon = \{0,1\}$ and $\sigma : \Upsilon \times \Upsilon \longrightarrow \mathbb{R}^+$ be defined by

$$\sigma(\iota,\kappa) = \begin{cases} 2, & if\ \iota = \kappa = 0, \\ 1, & otherwise. \end{cases}$$

Then (Υ,σ) is a metric-like space but it is not a partial metric space. Since $\sigma(0,0) > \sigma(0,1)$, then (Υ,σ) is not a partial metric space.

In 2013, Alghamdi et al. [5] introduced the concept of b-metric-like spaces and gave the following axioms.

Definition 2.3. [5] A b-metric-like space on a non-empty set Υ is a function $\sigma : \Upsilon \times \Upsilon \to [0,+\infty)$ such that for all $\iota,\kappa,z \in \Upsilon$ and a parameter $s \geq 1$ we have the following condition holds:

$(\sigma_b 1)$ if $\sigma(\iota,\kappa) = 0 \Rightarrow \iota = \kappa$;
$(\sigma_b 2)$ $\sigma(\iota,\kappa) = \sigma(\kappa,\iota)$; and
$(\sigma_b 3)$ $\sigma(x,z) \leq s[\sigma(\iota,\kappa) + \sigma(z,\kappa)]$.

Then σ_b is called a b-metric-like on Υ, so a pair (Υ,σ_b) is called a b-metric-like space.

The b-metric-like on Υ satisfies all of the conditions of metric except that $\sigma_b(\iota,\iota)$ may be positive for $\iota \in \Upsilon$. Hammad and De la Sen [22] proved that each b-metric-like σ_b on X generates a topology τ_{σ_b} on Υ whose base is the family of open balls

$$B_{\sigma_b}(\iota,\epsilon) = \Big\{\kappa \in \Upsilon : |\sigma_b(\iota,\kappa) - \sigma_b(\iota,\iota)| < \frac{\epsilon}{s}\Big\},$$

for all $\iota \in \Upsilon$ and $\epsilon > 0$. Then a sequence $\{\iota_n\}$ in metric-like space (Υ,σ_b) converges to a point $\iota \in \Upsilon$ if and only if

$$\lim_{n\to\infty} |\sigma_b(\iota_n,\iota) - \sigma_b(\iota,\iota)| < \frac{\epsilon}{s}.$$

The following are some properties of b-metric-like spaces.

Definition 2.4. [5] Let (Υ,σ_b) be a b-metric-like space, and let $\{\iota_n\}$ be a sequence of points of Υ. A point $\iota \in \Upsilon$ is said to be a limit of sequence $\{\iota_n\}$ if

$$\lim_{n\to\infty} \sigma_b(\iota_n,\iota) \quad = \quad \sigma_b(\iota,\iota),$$

and we say the sequence $\{\iota_n\}$ is convergent to ι and denote it by $\iota_n \to \iota$ as $n \to \infty$.

Definition 2.5. [5] Let (Υ,σ_b) be a b-metric-like space. Then,

(i) a sequence $\{\iota_n\}$ in (Υ,σ_b) is said to be a Cauchy sequence if $\lim_{m,n\to\infty} \sigma_b(\iota_n,\iota_m)$ exists and is finite.

(ii) a b-metric-like space (Υ,σ_b) is said to be complete if and only if every Cauchy sequence $\{\iota_n\} \subset \Upsilon$ converges to $\iota \in \Upsilon$ in such a way that

$$\lim_{m,n\to\infty} \sigma_b(\iota_n,\iota_m) \quad = \quad \sigma_b(\iota,\iota) = \lim_{n\to\infty} \sigma_b(\iota_n,\iota).$$

The limit of a sequence in a b-metric-like space need not be unique.

Lemma 2.1. [5] Let $\{\iota_n\}$ be a sequence in a b-metric-like space with parameter $s \geq 1$ such that

$$\sigma_b(\iota_n, \iota_{n+1}) \leq \lambda \sigma_b(\iota_{n-1}, \iota_n),$$

for some $0 < \lambda < \frac{1}{s}$, and each $n \in \mathbb{N}$. Then

$$\lim_{n,m\to\infty} \sigma_b(\iota_n, \iota_m) = 0.$$

Following are examples that accommodate all axioms of b-metric-like space.

Example 2.3. [5] Let $\Upsilon = \mathbb{R}^+ \cup \{0\}$. Define the function $\sigma_b : \Upsilon^2 \to [0, +\infty)$ by $\sigma_b(\iota, \kappa) = (\iota + \kappa)^2$, for all $\iota, \kappa \in \Upsilon$. Then (Υ, σ_b) is a b-metric-like space with parameter $s = 2$.

Example 2.4. [22] Let $\Upsilon = [0, +\infty)$ and $\sigma_b(\iota, \kappa) = \iota^2 + \kappa^2 + |\iota - \kappa|^2 \ \forall \iota, \kappa \in \Upsilon$. It is obvious that σ_b is a b-metric-like on Υ, with coefficient $s = 2$.

The following Lemma took from [43] shows that in general a b-metric-like space is not continuous.

Lemma 2.2. [43] Let (X, σ_b) be a b-metric-like space with parameter $s \geq 1$. Then

(1) $\sigma_b(\iota, \kappa) = 0$ then $\sigma_b(\iota, \iota) = \sigma_b(\kappa, \kappa) = 0$;

(2) If $\{\iota_n\}$ be a sequence in such a way that $\lim_{n\to\infty} \sigma_b(\iota_n, \iota_{n+1}) = 0$, then we have

$$\lim_{n\to\infty} \sigma_b(\iota_n, \iota_n) = \lim_{n\to\infty} \sigma_b(\iota_{n+1}, \iota_{n+1}) = 0;$$

(3) if $\iota \neq \kappa$, then $\sigma_b(\iota, \kappa) > 0$.

In 2012, Wardowski [41] gave the properties of F-contraction mapping as follows:

Let F be a function defined as $F : \mathbb{R}^+ \to \mathbb{R}$, which satisfies the following conditions:

(F1) F is strictly increasing i.e., for all $\alpha, \beta \in \mathbb{R}_+$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$;

(F2) For each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$;

(F3) There exists $k \in (0, 1)$ such that

$$\lim_{n\to\infty} (\alpha_n)^k F(\alpha_n) = 0.$$

Then the family of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ satisfying the conditions $(F1 - F3)$ is denoted by ∇_F.

Definition 2.6. [41] Let (Υ, d) be a metric space. A self-mapping μ on Υ is called an F-contraction mapping if there exists $F \in \nabla_F$ and $\tau \in \mathbb{R}^+$ such that

$$\forall \iota, \kappa \in \Upsilon,\ d(\mu\iota, \mu\kappa) > 0 \implies \tau + F(d(\mu\iota, \mu\kappa)) \leq F(d(\iota, \kappa)).$$

Wardowski [41] introduced a generalization of Banach contraction, which is as follows:

Theorem 2.1. [41] Let (Υ, d) be a complete metric space and $\mu : \Upsilon \longrightarrow \Upsilon$ be an F-contraction map. Then μ has a unique fixed point $\iota_0 \in \Upsilon$ and for every $\iota \in \Upsilon$ the sequence $\{\mu^n \iota\}_{n\in\mathbb{N}}$ converges to ι_0.

In 2020, Batra *et al.* [10] gave an extended version of Definition 2.6 using F-Kannan mapping as follows:

Definition 2.7. [10] Let F be a mapping satisfying $(F1)-(F3)$. A mapping $\mu : \Upsilon \to \Upsilon$ is said to be an F-Kannan mapping if the following condition holds:

(K1)

$$\mu\iota \neq \mu\kappa \Rightarrow \mu\iota \neq \iota \ or \ \mu\kappa \neq \kappa. \tag{4}$$

(K2) there exists $\Gamma > 0$ such that

$$\Gamma + F(d(\mu\iota, \mu\kappa)) \leq F\left[\frac{d(\iota, \mu\iota) + d(\kappa, \mu\kappa)}{2}\right], \tag{5}$$

for all $\iota, \kappa \in \Upsilon$, with $\mu\iota \neq \mu\kappa$.

Remark 2.2. [10] By properties of F-mapping, it follows that every F-Kannan mapping μ on a metric space (Υ, d), satisfies following condition:

$$d(\mu\iota, \mu\kappa)) \leq \frac{d(\iota, \mu\iota) + d(\kappa, \mu\kappa)}{2},$$

for every $\iota, \kappa \in \Upsilon$.

We refer to Batra *et al.* [10] as examples of such functions $F : \mathbb{R}^+ \longrightarrow \mathbb{R}$ which satisfy F-Kannan mappings.

In addition, Hammad and De la Sen [22] inspired by Zoto *et al.* [43] gave a generalized definition of (s, q)-Jaggi F-contraction type in a b-metric-like space as follows:

Definition 2.8. [22] Let μ be a self mapping on a b-metric-like space (Υ, σ_b) with parameter $s \geq 1$. Then the mapping μ is said to be a generalized (s, q)-Jaggi F-contraction type if there is $F \in \nabla_F$ and $\tau > 0$ such that

$$\begin{aligned} \sigma_b(\mu\iota, \mu\kappa) \ &> 0 \Rightarrow \\ \tau + F(s^q \sigma_b(\mu\iota, \mu\kappa)) \ &\leq \ F\Big(\alpha \frac{d(\iota, \mu\iota).d(\kappa, \mu\kappa)}{d(\iota, \kappa)} + \beta d(\iota, \kappa)\Big), \end{aligned} \tag{6}$$

for all $\iota, \kappa \in \Upsilon$ such that $\iota \neq \kappa$, and for some $\alpha, \beta \geq 0$ with $\alpha + \beta < 1$.

Theorem 2.2. [22] Let (Υ, σ_b) be a 0-σ_b- complete metric-like space with a coefficient $s \geq 1$ and μ be a self mapping satisfying the generalized almost (s, q)-Jaggi F-contraction-type. Then μ has a unique fixed point whenever F or μ is continuous.

After that, Joshi *et al.* [24] introduced the concept of ϕ-F-contraction in a b-metric-like space as follows: Let Φ be the set of functions $\phi : [0, \infty) \to [0, \infty)$ such that

(1) ϕ is monotonic increasing that is,

$$t_1 \leq t_1 \implies \phi(t_1) \leq \phi(t_1),$$

(2) ϕ is continuous and

$$\phi(t) \leq (t),$$

for each $t > 0$.

Let ψ denotes the set of all continuous functions $\psi : [0,\infty) \to [0,\infty)$.

Definition 2.9. [24] Let (Υ, σ_b) be a b-metric-like space. A self mapping $\mu : \Upsilon \to \Upsilon$ is said to be a generalized ϕ-F-contraction for some $F \in \nabla_F$ and

$$\begin{aligned} &\sigma_b(\mu\iota, \mu\kappa) > 0 \Rightarrow \\ \psi(\sigma_b(\iota,\kappa)) + F(\sigma_b(\mu\iota,\mu\kappa)) \quad &\leq \quad F\Big(\phi(\alpha\sigma_b(\iota,\kappa) + \beta\frac{\sigma_b(\iota,\mu\kappa)}{2s} + \gamma\frac{\sigma_b(\kappa,\mu\iota)}{2s})\Big), \end{aligned}$$

for all $\iota, \kappa \in \Upsilon$, where $\alpha, \beta, \gamma \in [0,1]$ such that $\alpha+\beta+\gamma \leq 1$, $\phi \in \Phi$ and $\psi \in \Psi$.

Theorem 2.3. [24] Let (Υ, σ_b) be a complete b-metric-like space with a coefficient $s \geq 1$ and μ be a continuous generalized ϕ-F-contraction. If $\sigma_b(\mu\iota,\mu\iota) \leq \sigma_b(\iota,\iota)$, for all $\iota \in \Upsilon$, then μ has a unique fixed point Υ.

Furthermore, Abbas and Jungck [1] gave the following definition for a unique common fixed point notion:

Definition 2.10. [1]

(i) Let μ and ν be self maps on a set Υ. If $z = \mu\iota = \nu\iota$ for some ι in Υ, then ι is called a coincidence point of μ and ν, and z is called a point of coincidence of μ and ν.

(ii) Let μ and ν be weakly compatible self maps of a set Υ. If μ and ν have a unique point of coincidence $z = \mu\iota = \nu\iota$, then z is the unique common fixed point of μ and ι.

Matkowski [32] established the concept of a nondecreasing function as follows:

Lemma 2.3. [32] Suppose $\gamma : [0,\infty) \to [0,\infty)$ is non dencreasing. Then for every $t > 0$,

$$\lim_{n\to\infty} \gamma^n(t) = 0 \Leftrightarrow \gamma(t) < t.$$

The following example is a right continuous function $\gamma : [0,\infty) \to [0,\infty)$ such that $\gamma(t) < t$ for $t > 0$, $\lim_{n\to\infty} \gamma^n(t) = 0$.

Example 2.5. [32] Let $\Upsilon = [0,\infty)$, then

$$\gamma(t) = \begin{cases} t, & t > 0, \\ \frac{t}{1+nt}, & t \in (1,\infty), \end{cases}$$

for all $\iota, \kappa, t \in \Upsilon$ and $n \in \mathbb{R}$.

3. Main Result

Motivated by Batra *et al.* [10], we state an extended version of F-Kannan mapping in a b-metric-like space.

To develop our first main result, we will provide an extension of Definition 2.7 in b-metric-like spaces.

Definition 3.1. Let F be a mapping satisfying $(F1) - (F3)$. A self-mapping $\mu : \Upsilon \to \Upsilon$ is said to be an (γ, s, q)-F-Kannan mapping if the following conditions hold:

(KB1)

$$\mu\iota \neq \mu\kappa \Rightarrow \mu\iota \neq \iota, \mu\kappa \neq \kappa. \tag{7}$$

(KB2) there exists $\Gamma > 0$ such that

$$\Gamma + F(\gamma(s^q\sigma_b(\mu\iota, \mu\kappa))) \leq F\left[\frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}\right], \tag{8}$$

for all $\iota, \kappa \in \Upsilon$, with $\mu\iota \neq \mu\kappa$, for some $\gamma > 0, q > 1$.

We give the following examples in the context of (γ, s, q)-F-Kannan mappings in a b-metric-like space:

Example 3.1. Let $F_1 : \mathbb{R}^+ \to \mathbb{R}$ be defined as $F_1(z) = \ln(z)$. Then clearly, $(F1) - (F3)$ are satisfied by $F_1(z)$. In fact $(F3)$ holds for every $k \in (0, 1)$. Moreover, Equation (8) above takes the form:

$$\gamma(s^q\sigma_b(\mu\iota, \mu\kappa)) \leq e^{-\Gamma}\left[\frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}\right], \tag{9}$$

for all $\iota, \kappa \in \Upsilon$ with $\mu\iota \neq \mu\kappa$ and $\gamma > 0$.

Thus, if $\mu : \Upsilon \to \Upsilon$ is a (γ, s, q)-Kannan mapping with constant $k \in (0, 1)$ satisfying

$$\gamma(s^q\sigma_b(\mu\iota, \mu\kappa)) \leq k\left[\frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}\right] \tag{10}$$

for every $\iota, \kappa \in \Upsilon$, then it also satisfies (9) and (8) with $\Gamma = ln\frac{1}{k}$.

Example 3.2. Let $F_2 : \mathbb{R}^+ \to \mathbb{R}$ be defined as $F_2(z) = \ln(z) + z, z > 0$, then $(F1) - (F3)$ are satisfied by $F_2(z)$. Equation (8) above takes the form

$$\frac{\gamma(s^q\sigma_b(\mu\iota, \mu\kappa))}{(\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa))/2} e^{\gamma(s^q\sigma_b(\mu\iota, \mu\kappa))} - \left\{\frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}\right\} \leq e^{-\Gamma}, \tag{11}$$

for all $\iota, \kappa \in \Upsilon$ with $\mu\iota \neq \mu\kappa$.

Remark 3.1. By properties of an F-mapping, it follows that every (γ, s, q)-F-Kannan mapping μ on a b-metric-like space (Υ, σ_b), satisfies following condition:

$$\gamma(s^q\sigma_b(\mu\iota, \mu\kappa)) \leq \frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}, \tag{12}$$

for every $\iota, \kappa \in \Upsilon$.

Proof. From $(KB2)$, we have

$$\Gamma + F(\gamma(s^q\sigma_b(\mu\iota, \mu\kappa))) \leq F\left[\frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}\right]. \tag{13}$$

By the continuity properties of F, we get

$$F(\gamma(s^q\sigma_b(\mu\iota, \mu\kappa))) < F\left[\frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}\right]. \tag{14}$$

Consequently, we get

$$\gamma(s^q\sigma_b(\mu\iota, \mu\kappa)) < \frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}, \tag{15}$$

which is a contradiction. Therefore, we have

$$\gamma(s^q\sigma_b(\mu\iota, \mu\kappa)) \leq \frac{\sigma_b(\iota, \mu\iota) + \sigma_b(\kappa, \mu\kappa)}{2}. \tag{16}$$

Hence, the proof is completed. □

Now, we prove the following theorem.

Theorem 3.1. Let (Υ, σ_b) be a complete b-metric-like space and $\mu : \Upsilon \to \Upsilon$ be F-Kannan mapping satisfying $(F1) - (F3)$. Assume that the following conditions hold:

(i) $(KB1)$ and $(KB2)$ hold,

(ii) μ is continuous at some point $z \in \Upsilon$,

(iii) there exists a a nondecreasing function γ with some point $\iota_0 \in \Upsilon$ such that the sequence $\{\mu^n \iota_n\}_{n\geq 0}$ is convergent to z.

Then z is a unique fixed point of μ. Further $\sigma_b(z, z) = 0$.

Proof. Let $\iota_0 \in \Upsilon$ be arbitrary. Define a sequence $\{\iota_n\}$ in Υ by $\iota_n = \mu\iota_{n-1}$ for all $n \in \mathbb{N}$. Now, denote $\alpha_n = \sigma_b(\iota_n, \iota_{n+1})$ for all $n \in \mathbb{N} \cup \{0\}$ and γ a a non-decreasing function. If $\iota_n = \iota_{n+1}$, then $\alpha_n = 0$. If not, such that $\iota_n \neq \iota_{n+1}$, that is, $\mu\iota_{n+1} \neq \mu\iota_n$ for all $n \in \mathbb{N}$, using Equation 8 with $x = \iota_{n-1}$ and $\kappa = \iota_n$, we get

$$\begin{aligned}
\Gamma + F(\alpha_n) &\leq \Gamma + F(\gamma(s^q \alpha_n)) = \Gamma + F(\gamma(s^q \sigma_b(\iota_n, \iota_{n+1})))\\
&= \Gamma + F(\gamma(s^q \sigma_b(\mu\iota_{n-1}, \mu\iota_n)))\\
&\leq F\left[\frac{\sigma_b(\iota_{n-1}, \mu x_{n-1}) + \sigma_b(\iota_n, \mu\iota_n)}{2}\right],\\
&\leq F\left[\frac{\sigma_b(\iota_{n-1}, \iota_n) + \sigma_b(\iota_n, \iota_{n+1})}{2}\right].
\end{aligned}$$

Since F is strictly increasing, using Remark 3.1, and the property of γ we get

$$\begin{aligned}
\gamma(s^q \sigma_b(\mu\iota_{n-1}, \mu\iota_n)) &\leq \frac{\sigma_b(\iota_{n-1}, \mu\iota_{n-1}) + \sigma_b(\iota_n, \mu\iota_n)}{2},\\
\sigma_b(\iota_n, \iota_{n+1}) &\leq \frac{\sigma_b(\iota_{n-1}, \iota_n) + \sigma_b(\iota_n, \iota_{n+1})}{2}.
\end{aligned}$$

This leads to

$$\begin{aligned}
\alpha_n < \gamma(s^q \alpha_n) &< \frac{\sigma_b(\iota_{n-1}, \mu\iota_{n-1}) + \sigma_b(\iota_n, \mu\iota_n)}{2},\\
\Rightarrow 2\sigma_b(\iota_n, \iota_{n+1},) &\leq \sigma_b(\iota_{n-1}, \iota_n) + \sigma_b(\iota_n, \iota_{n+1}),\\
\Rightarrow 2\sigma_b(\iota_n, \iota_{n+1}) - \sigma_b(\iota_n, \iota_{n+1}) &\leq \sigma_b(\iota_{n-1}, \iota_n),\\
\Rightarrow \sigma_b(\iota_n, \iota_{n+1}) &\leq \sigma_b(\iota_{n-1}, \iota_n),
\end{aligned}$$

and hence

$$\sigma_b(\iota_n, \iota_{n+1}) < \sigma_b(\iota_{n-1}, \iota_n).$$

Consequently,

$$\Gamma + F(\sigma_b(\iota_n, \iota_{n+1})) \leq F(\sigma_b(\iota_{n-1}, \iota_n)),$$

which implies that

$$F(\sigma_b(\iota_n, \iota_{n+1})) \leq F\Big[\sigma_b(\iota_{n-1}, \iota_n)\Big] - \Gamma. \tag{17}$$

Similarly, for $\iota = \iota_n$ and $\kappa = \iota_{n+1}$ using (8) we obtain

$$F(\sigma_b(\mu\iota_n, \mu\iota_{n+1})) \leq F\left[\frac{\sigma_b(\iota_n, \mu\iota_n) + \sigma_b(\iota_{n+1}, \mu\iota_{n+1})}{2}\right] - \Gamma,$$

which implies that

$$F(\sigma_b(\iota_{n+1},\iota_{n+2})) \leq F[\sigma_b(\iota_n,\iota_{n+1})]-\Gamma. \tag{18}$$

Using (17) in (18) we get

$$F(\sigma_b(\iota_{n+1},\iota_{n+2})) \leq F[\sigma_b(\iota_{n-1},\iota_n)]-2\Gamma. \tag{19}$$

Now, by induction for all $n=1,2,\dots(n\in\mathbb{N})$ we deduce that

$$F(\sigma_b(\iota_n,\iota_{n+1})) \leq F[\sigma_b(\iota_{n-1},\iota_n)]-n\Gamma. \tag{20}$$

Letting $n\to\infty$ in (20) and using condition $(F2)$ of F results to

$$\begin{aligned}\lim_{n\to\infty} F(\sigma_b(\iota_n,\iota_{n+1})) &= -\infty.\\ \Rightarrow \lim_{n\to\infty}\sigma_b(\iota_n,\iota_{n+1}) &= 0.\end{aligned}$$

By $(F3)$ of F, there exists $k\in(0,1)$ such that

$$\lim_{n\to\infty}(\alpha_n)^k F(\alpha_n)=0. \tag{21}$$

From (20), for every $n\in\mathbb{N}$, we have

$$\begin{aligned}(\alpha_n)^k F(\alpha_n) &\leq \cdots \leq (\alpha_n)^k F(\alpha_{n-1})-n\Gamma(\alpha_n)^k,\\ (\alpha_n)^k F(\alpha_n)-(\alpha_n)^k F(\alpha_{n-1}) &\leq -n\Gamma(\alpha_n)^k,\\ (\alpha_n)^k[F(\alpha_n)-F(\alpha_{n-1})] &\leq -n\Gamma(\alpha_n)^k \leq 0.\end{aligned} \tag{22}$$

On taking limit as $n\to\infty$ in (22), we get

$$\lim_{n\to\infty} n(\alpha_n)^k=0. \tag{23}$$

From (23), there exists $n_1\in\mathbb{N}$ such that $n(\alpha_n)^k\leq 1$, for all $n\geq n_1$, which follows that

$$\alpha_n\leq n^{-\frac{1}{k}}, \forall n\geq n_1. \tag{24}$$

Therefore, $\sum_{n=m}^{\infty}\sigma_b(\iota_n,\iota_{n+1})$ converges.

Now, we prove that $\{x_n\}$ is a Cauchy sequence. Consider $n,m\in n_1$ such that $m,n\geq n_1$. By (σ_b3) of Definition 2.3 and Lemma 2.1, we have

$$\begin{aligned}\sigma_b(\iota_n,\iota_m) &\leq s\Big[\sigma_b(\iota_n,\iota_{n+1})+\sigma_b(\iota_{n+1},\iota_m)\Big]\\ &= s\sigma_b(\iota_n,\iota_{n+1})+s\sigma_b(\iota_{n+1},\iota_m)\\ &\leq s\sigma_b(\iota_n,\iota_{n+1})+s^2\Big[\sigma_b(\iota_{n+1},\iota_{n+2})+\sigma_b(\iota_{n+2},\iota_m)\Big],\\ &\leq s\sigma_b(\iota_n,\iota_{n+1})+s^2\sigma_b(\iota_{n+1},\iota_{n+2})+s^3\sigma_b(\iota_{n+2},\iota_{n+3})+s^4\sigma_b(\iota_{n+3},\iota_{n+4}),\end{aligned}$$

$$
\begin{aligned}
&\leq s\sigma_b(\iota_n, \iota_{n+1}) + s^2\sigma_b(\iota_{n+1}, \iota_{n+2}) + s^3\sigma_b(\iota_{n+2}, \iota_{n+3}) + \\
&\quad s^4\sigma_b(\iota_{n+3}, \iota_{n+4}) + \ldots, \\
&\leq s\alpha_n + s^2\alpha_{n+1} + s^3\alpha_{n+2} + s^4\alpha_{n+3} + \ldots, \\
&\leq s\alpha_n[1 + s\alpha_n + s^2\alpha_{n+1} + s^3\alpha_{n+2} + \ldots], \\
&\leq s\alpha_n[1 + s\alpha_n + s^2\alpha_{n+1} + s^3\alpha_{n+2} + \ldots], \\
&\leq \frac{s\alpha_n}{1 - s\alpha_n}, \\
&\leq \sum_{i=n}^{m-1} i^{-\frac{1}{k}}.
\end{aligned}
$$

Since the series $\sum_{i=n}^{m-1} i^{-\frac{1}{k}}$ converges, which implies that

$$\lim_{n\to\infty} \sigma_b(\iota_n, \iota_m) = 0.$$

Hence $\{\iota_n\}, \forall n \in \mathbb{N}$ is a Cauchy sequence in Υ.

Consider the following two distinct fixed points of $\{\iota_n\}$ in Υ such that

$$\mu^n\iota = \begin{cases} z, & \textit{when } n \textit{ is odd}, \\ w, & \textit{when } n \textit{ is even}. \end{cases}$$

The completeness of Υ ensures the existence of $z \in \Upsilon$ such that $\mu^n x \to z$ as $n \to \infty$. By ($\sigma_b 3$) we have,

$$
\begin{aligned}
\sigma_b(z, \mu z) &\leq s\Big[\sigma_b(z, \mu^{n+1}\iota) + \sigma_b(\mu^{n+1}\iota, \mu z)\Big], \\
&\leq s\sigma_b(z, \mu^{n+1}\iota) + s\sigma_b(\mu^{n+1}\iota, \mu z), \\
&\leq s\sigma_b(z, \mu z) + s\sigma_b(\mu z, \mu z), \\
&\leq s\sigma_b(z, \mu z),
\end{aligned}
$$

it follows that $\sigma_b(z, \mu z) = 0$. By continuity of μ, we have

$$z = \lim_{n\to\infty} \mu^n\iota = \lim_{n\to\infty} \mu^{n+1}\iota = \lim_{n\to\infty} \mu\iota_n = \mu z.$$

Similarly, if w be another element of Υ such that $\mu^n\iota = w$, w is a fixed point of μ. Using ($\sigma_b 3$) we get,

$$
\begin{aligned}
\sigma_b(w, \mu w) &\leq s\Big[\sigma_b(w, \mu^{n+1}\iota) + \sigma_b(\mu^{n+1}\iota, \mu w)\Big], \\
&\leq s\sigma_b(w, \mu^{n+1}\iota) + s\sigma_b(\mu^{n+1}\iota, \mu w), \\
&\leq s\sigma_b(w, \mu w) + s\sigma_b(\mu w, \mu w), \\
&\leq s\sigma_b(w, \mu w),
\end{aligned}
$$

it follows that $\sigma_b(w, \mu w) = 0$. By continuity of μ, we have

$$w = \lim_{n\to\infty} \mu^n\iota = \lim_{n\to\infty} \mu^{n+1}\iota = \lim_{n\to\infty} \mu\iota_n = \mu w,$$

which is a contradiction. We conclude that $z \neq w$. Hence, μ has a fixed point in Υ which is not unique. Hence, the proof is completed. □

Example 3.3. Let $\Upsilon = [0,\infty] \cap \mathbb{Q}$ with b-metric $\sigma_b(\iota,\kappa) = (\iota+\kappa)^2$ for all $\iota,\kappa \in \Upsilon$. Consider the function $\mu : \Upsilon \longrightarrow \Upsilon$ given by

$$\mu\iota = \begin{cases} \sqrt{\dfrac{\iota}{4}}, & \iota \in \left[0, \dfrac{1}{2}\right], \\ \\ \sqrt{2-\dfrac{1}{\iota}}, & \iota \in [1,2]. \end{cases}$$

Take $F_2(z) = \ln z + z$, $q = 2, s = 2$ and $\Gamma = \frac{1}{s}, \gamma(t) = \frac{t}{1+\sqrt{2t}}$. We shall prove that T satisfy the condition (8). Here, one may notice that $\sigma_b(\iota_n,\iota_{n+1}) \in \Upsilon$, for all $n \in \mathbb{N}_0$ and there exists an integer $N \in \mathbb{N}_0$ such that $\iota_n = \iota \in \{0, \frac{1}{2}, 1, 2\}$ for $n \leq \mathbb{N}$. Therefore Υ is μ-closed.

Now, for $\iota \in [0, \dfrac{1}{2}]$ and $\kappa \in [1,2]$, we have

$$\begin{aligned} \sigma_b(\mu\iota,\mu\kappa) &= \sigma_b\left(\sqrt{\frac{\iota}{4}}, \sqrt{2-\frac{1}{\kappa}}\right), \\ &= \left(\sqrt{\frac{\iota}{4}} + \sqrt{2-\frac{1}{\kappa}}\right)^2, \\ &= \frac{\iota\kappa + 2\iota\kappa\sqrt{\frac{2\kappa-1}{\kappa}} + 8\kappa - 4}{4\kappa}. \end{aligned}$$

and

$$\begin{aligned} \sigma_b(\iota,\mu\iota) &= \sigma_b\left(\iota, \sqrt{\frac{\iota}{4}}\right), \\ &= \left(\iota + \sqrt{\frac{\iota}{4}}\right)^2, \\ &= \frac{4\iota^2+\iota}{4} + 2\iota\sqrt{\frac{\iota}{4}}, \\ &= \frac{4\iota^2 + \iota + 4\iota\sqrt{\iota}}{4}. \end{aligned}$$

$$\begin{aligned} \sigma_b(\kappa,\mu\kappa) &= \sigma_b\left(\kappa, \sqrt{2-\frac{1}{\kappa}}\right), \\ &= \left(y + \sqrt{2-\frac{1}{\kappa}}\right)^2, \\ &= \frac{\kappa^3 + 2\kappa - 1 + 2\kappa^2\sqrt{\frac{2\kappa-1}{\kappa}}}{\kappa}. \end{aligned}$$

By (11) and Let $P = \frac{\iota\kappa+2\iota\kappa\sqrt{\frac{2\kappa-1}{\kappa}}+8\kappa-4}{y}$ and $Q = \frac{1}{2}\left(\frac{4\iota^2+\iota+4\iota\sqrt{\iota}}{4} + \frac{\kappa^3+2\kappa-1+2\kappa^2\sqrt{\frac{2\kappa-1}{\kappa}}}{\kappa}\right)$, we get

$$\frac{\gamma(P)}{Q} e^{\gamma(P)-Q} \leq e^{-\Gamma}.$$

Next, for $\iota,\kappa \in [0, \dfrac{1}{2}]$, we have

$$\begin{aligned}
\sigma_b(\mu\iota,\mu\kappa) &= \sigma_b\left(\sqrt{\frac{\iota}{4}},\sqrt{\frac{\kappa}{4}}\right),\\
&= \left(\sqrt{\frac{\iota}{4}}+\sqrt{\frac{\kappa}{4}}\right)^2,\\
&= \frac{1}{4}(\iota+\kappa+2\sqrt{\iota\kappa}),
\end{aligned}$$

and

$$\begin{aligned}
\sigma_b(\iota,\mu\iota) &= \sigma_b\left(\iota,\sqrt{\frac{\iota}{4}}\right),\\
&= \left(\iota+\sqrt{\frac{\iota}{4}}\right)^2,\\
&= (\iota^2+\iota\sqrt{\iota}+\frac{\iota}{4}),
\end{aligned}$$

$$\begin{aligned}
\sigma_b(\kappa,\mu\kappa) &= \sigma_b\left(\kappa,\sqrt{\frac{\kappa}{4}}\right),\\
&= \left(\kappa+\sqrt{\frac{\kappa}{4}}\right)^2,\\
&= (\kappa^2+\kappa\sqrt{\kappa}+\frac{\kappa}{4}).
\end{aligned}$$

By (11), let $F=\iota+\kappa+2\sqrt{\iota\kappa}$ and $H=\frac{4\iota^2+4\kappa^2+4\iota\sqrt{\iota}+4\kappa\sqrt{\kappa}+\iota+\kappa}{8}$, we get

$$\frac{\gamma(F)}{H}e^{\gamma(F)-H}\leq e^{-\Gamma}.$$

Similarly, for $\iota,\kappa\in[1,2]$, we have

$$\begin{aligned}
\sigma_b(\mu\iota,\mu\kappa) &= \sigma_b\left(\sqrt{2-\frac{1}{\iota}},\sqrt{2-\frac{1}{\kappa}}\right),\\
&= \left(\sqrt{2-\frac{1}{\iota}}+\sqrt{2-\frac{1}{\kappa}}\right)^2,\\
&= \frac{4\iota\kappa-\kappa-\iota}{\iota\kappa}+2\sqrt{\frac{4\iota\kappa-2\iota-2\kappa+1}{\iota\kappa}},\\
&= \frac{4\iota\kappa-\iota-\kappa+2\iota\kappa\sqrt{\frac{4\iota\kappa-2\iota-2\kappa+1}{\iota\kappa}}}{\iota\kappa},
\end{aligned}$$

and

$$\begin{aligned}
\sigma_b(\iota,\mu\iota) &= \sigma_b\left(\iota,\sqrt{2-\frac{1}{\iota}}\right),\\
&= \left(\iota+\sqrt{2-\frac{1}{\iota}}\right)^2,
\end{aligned}$$

$$= \frac{\iota^3 + 2\iota^2\sqrt{2-\frac{1}{\iota}} + 2\iota - 1}{\iota},$$

$$\begin{aligned}\sigma_b(\kappa, \mu\kappa) &= \sigma_b\Bigg(\kappa, \sqrt{2-\frac{1}{\kappa}}\Bigg),\\ &= \Bigg(\kappa + \sqrt{2-\frac{1}{\kappa}}\Bigg)^2,\\ &= \frac{\kappa^3 + 2\kappa^2\sqrt{2-\frac{1}{\kappa}} + 2\kappa - 1}{\kappa},\end{aligned}$$

By (11) and let $L = \frac{4(4\iota\kappa - \iota - \kappa + 2\iota\kappa\sqrt{\frac{4\iota\kappa - 2\iota - 2\kappa + 1}{\iota\kappa}}}{\iota\kappa}$ and $M = \frac{\iota^3\kappa + \kappa^3\iota + 2\iota^2\kappa\sqrt{2-\frac{1}{\iota}} + 2\iota\kappa^2\sqrt{2-\frac{1}{\kappa}} + 4\iota\kappa - \kappa - \iota}{2\iota\kappa}$, we get

$$\frac{\gamma(L)}{M} e^{\gamma(L) - M} \leq e^{-\Gamma}.$$

Thus our claim is satisfied and all the conditions imposed in Theorem 3.1 are also satisfied. Hence, μ has two unique distinct fixed points, that are $\iota = 0$ and $\iota = 1$.

Our second main result proves a common fixed point theorem for (γ, s, q)- F-contraction mappings in b-metric-like space.

First, we will provide the extension of Definition 2.9 to two self maps in b-metric-like space as follows:

Definition 3.2. Let (Υ, σ_b) be a b-metric-like space. The mappings $\mu, \nu : \Upsilon \to \Upsilon$ is said to be generalized (γ, s, q)-F-Reich type contraction if $F \in \nabla_F$ and

$$\begin{aligned}&\sigma_b(\mu\iota, \mu\kappa) > 0 \Rightarrow\\ \Gamma + \gamma(F(s^q\sigma_b(\mu\iota, \mu\kappa))) &\leq F\Big(\eta\sigma_b(\nu\iota, \nu\kappa) + \zeta\frac{\sigma_b(\nu\iota, \mu\kappa)}{2s} + \xi\frac{\sigma_b(\nu\kappa, \mu\iota)}{2s}\Big),\end{aligned} \tag{25}$$

for all $\iota, \kappa \in \Upsilon$, where $\eta, \zeta, \xi \in [0, 1]$ such that $\eta + \zeta + \xi \leq 1$, and some $q > 1$.

Theorem 3.2. Let (Υ, σ_b) be a complete b-metric-like space with a coefficient $s \geq 1$ and $\mu, \nu : \Upsilon \to \Upsilon$ be two self mapping. If the pair (μ, ν) are continuous generalized (γ, s, q)-F-Reich type contraction pair, then μ and ν has a unique common fixed point in Υ.

Proof. Let $\iota_0 \in \Upsilon$ be arbitrary. Assume that $\mu(\Upsilon) \subseteq \nu(\Upsilon)$. We construct a sequence $\{\iota_n\}$ by $\iota_n = \nu\iota_n = \mu\iota_{n-1}$ for all $n \in \mathbb{N}$ and $\iota_{n+1} = \nu\iota_{n+1} = \mu\iota_n$ for all $n \in \mathbb{N}$. Now, denote $\alpha_n = \sigma_b(\iota_n, \iota_{n+1})$ for all $n \in \mathbb{N} \cup \{0\}$. If $\iota_n = \iota_{n+1}$, then $\alpha_n = 0$. If not, such that $\iota_n \neq \iota_{n+1}$, that is, $\nu\iota_{n+1} \neq \mu\iota_n$ for all $n \in \mathbb{N}$. Let $\sigma_b(x_n, x_{n+1}) > 0$, using (25) with $\iota = \iota_{n-1}$ and $\kappa = \iota_n$, we have

$$\begin{aligned}\Gamma + F(\alpha_n) &\leq \Gamma + \gamma(F(s^q\alpha_n)) = \Gamma + \gamma(F(s^q\sigma_b(\iota_n, \iota_{n+1})))\\ &= \Gamma + \gamma(F(s^q\sigma_b(\mu\iota_{n-1}, \mu\iota_n)))\\ &\leq F\Big(\eta\sigma_b(\nu\iota_{n-1}, \nu\iota_n) + \zeta\frac{\sigma_b(\nu\iota_{n-1}, \mu\iota_n)}{2s} + \xi\frac{\sigma_b(\nu\iota_n, \mu\iota_{n-1})}{2s}\Big),\\ &\leq F\Big(\eta\sigma_b(\iota_{n-1}, \iota_n) + \zeta\frac{\sigma_b(\iota_{n-1}, \iota_{n+1})}{2s} + \xi\frac{\sigma_b(\iota_n, \iota_n)}{2s}\Big),\end{aligned}$$

$$\begin{aligned}&\leq F\Big(\eta\sigma_b(\iota_{n-1},\iota_n)+\zeta s\frac{(\sigma_b(\iota_{n-1},\iota_n)+\sigma_b(\iota_n,\iota_{n+1}))}{2s}+\xi\frac{\sigma_b(\iota_n,\iota_n)}{2s}\Big),\\ &\leq F\Big(\eta\sigma_b(\iota_{n-1},\iota_n)+\zeta\frac{\sigma_b(\iota_{n-1},\iota_n)}{2}+\zeta\frac{\sigma_b(\iota_n,\iota_{n+1})}{2}\Big),\end{aligned}$$

By $(F1)$ property of F and γ, we get

$$\begin{aligned}\sigma_b(\iota_n,\iota_{n+1}) &< \eta\sigma_b(\iota_{n-1},\iota_n)+\zeta\frac{\sigma_b(\iota_{n-1},\iota_n)}{2}+\zeta\frac{\sigma_b(\iota_n,\iota_{n+1})}{2},\\ \frac{2-\zeta}{2}\sigma_b(\iota_n,\iota_{n+1}) &< \frac{\zeta+2\eta}{2}\sigma_b(\iota_{n-1},\iota_n),\\ \sigma_b(\iota_n,\iota_{n+1}) &< \frac{\zeta+2\eta}{2-\zeta}\sigma_b(\iota_{n-1},\iota_n),\end{aligned}$$

taking $\frac{\zeta+2\eta}{2-\zeta}<1$, implies that

$$\sigma_b(\iota_n,\iota_{n+1}) < \sigma_b(\iota_{n-1},\iota_n).$$

Consequently,

$$\begin{aligned}\Gamma+F(\sigma_b(\iota_n,\iota_{n+1})) &\leq F(\sigma_b(\iota_{n-1},\iota_n)).\\ \Rightarrow F(\sigma_b(\iota_n,\iota_{n+1})) &\leq F(\sigma_b(\iota_{n-1},\iota_n))-\Gamma.\end{aligned}$$

Repeating this process, leads to

$$F(\sigma_b(\iota_n,\iota_{n+1})) \leq F(\sigma_b(\iota_0,\iota_1))-n\Gamma. \tag{26}$$

Taking $n\to\infty$, (26) gives

$$\begin{aligned}\lim_{n\to\infty}F(\sigma_b(\iota_n,\iota_{n+1})) &= -\infty,\\ \Rightarrow\lim_{n\to\infty}\sigma_b(\iota_n,\iota_{n+1}) &= 0.\end{aligned}$$

The other steps for convergence follow similar proof of Theorem 3.1.

Since $\nu(\Upsilon)$ is complete, there exists a point $z\in\Upsilon$ such that $\nu\iota_n\to z$ as $n\to\infty$. Now, we prove that z is a fixed point of ν.

On contrary, suppose that $z\neq\nu z$. Using triangle inequality we have

$$\begin{aligned}\sigma_b(z,\nu z) &\leq s[\sigma_b(z,\nu\iota_{n+1})+\sigma_b(\nu\iota_{n+1},\nu z)],\\ \sigma_b(z,\nu z) &= s\sigma_b(z,\nu\iota_{n+1})+s\sigma_b(\nu\iota_{n+1},\nu z),\end{aligned}$$

taking limit $n\to\infty$ in the above inequality, we obtains

$$\begin{aligned}\sigma_b(z,\nu z) &\leq s\sigma_b(z,z)+s\sigma_b(z,\nu z),\\ \sigma_b(z,\nu z)-s\sigma_b(z,\nu z) &\leq s\sigma_b(z,z),\\ (1-s)\sigma_b(z,\nu z) &\leq s\sigma_b(z,z),\\ \sigma_b(z,\nu z) &= 0,\end{aligned}$$

which is a contradiction. Hence $z=\nu z$.

Next, we shall show that z is a unique common fixed point of μ and ν. We claim that $\sigma_b(z,\nu z)>0$, then $\sigma_b(z,\nu z)>0$ for all $n\in\mathbb{N}$. Applying contraction condition (25) with $\iota=\iota_n$

and $\kappa = z$, we get

$$\begin{aligned}\Gamma + F(\alpha_n) &\leq \Gamma + F(s^q\alpha_n) = \Gamma + F(s^q\sigma_b(\iota_n, \iota_{n+1}))\\ &= \Gamma + F(s^q\sigma_b(\nu\iota_n, \mu z))\\ &\leq F\Big(\eta\sigma_b(\nu\iota_n, \nu z) + \zeta\frac{\sigma_b(\nu\iota_n, \mu z)}{2s} + \xi\frac{\sigma_b(\nu z, \mu\iota_n)}{2s}\Big),\\ &\leq F\Big(\eta\sigma_b(\nu\iota_n, \nu z) + \zeta\frac{\sigma_b(\nu\iota_n, \mu z)}{2s} + \xi\frac{\sigma_b(\nu z, \nu\iota_{n+1})}{2s}\Big),\\ &\leq F\Big(\eta\sigma_b(z, \nu z) + \zeta\frac{\sigma_b(z, \mu z)}{2s} + \xi\frac{\sigma_b(\nu z, z)}{2s}\Big).\end{aligned}$$

By $(F1)$ property of F and taking $n \to \infty$, we have

$$\begin{aligned}\sigma_b(z, \mu z) &< \eta\sigma_b(z, \nu z) + \zeta\frac{\sigma_b(z, \mu z)}{2s} + \xi\frac{\sigma_b(\nu z, z)}{2s},\\ \sigma_b(z, \mu z) &< \frac{2s\eta + \zeta}{2s - \xi}\sigma_b(z, \nu z) < \sigma_b(z, \nu z),\end{aligned}$$

consequently

$$\Gamma + \sigma_b(z, \mu z) \leq \sigma_b(z, \nu z).$$

For $\sigma_b(z, \mu z) = 0$ and $\sigma_b(z, \nu z) = 0$, implies that $\Gamma \leq 0$ which is a contradiction. Hence, $z = \mu z = \nu z$ is a unique common fixed point of μ and ν. □

The following example validates Theorem 3.2.

Example 3.4. Let $\Upsilon = [0, \infty] \cap \mathbb{Q}$ with b-metric like $\sigma_b(\iota, \kappa) = \iota^2 + \kappa^2 + |\iota - \kappa|^2$ for all $\iota, \kappa \in \Upsilon$. Consider the function $\mu, \nu : \Upsilon \longrightarrow \Upsilon$ defined by

$$\mu\iota = \frac{\sqrt{\iota}}{2} \text{ and } \nu(\iota) = \frac{\iota e^{\iota}}{4}.$$

Take $F_1(z) = \ln z$, $q = 2, s = 2$ and $\Gamma = \frac{1}{s}$, $\gamma(t) = \frac{t}{1+40t}$ with $\eta = \frac{1}{4}, \zeta = \frac{1}{6}, \xi = \frac{1}{3}$. We shall prove that μ and ν satisfy the condition (25).

We begin with simple calculations of the following .b-metrics-like

For $\iota, \kappa \in [0, \infty)$, we have

$$\begin{aligned}\sigma_b(\mu\iota, \mu\kappa) &= \sigma_b\left(\frac{\sqrt{\iota}}{2}, \frac{\sqrt{\kappa}}{2}\right),\\ &= \left(\frac{\sqrt{\iota}}{2}\right)^2 + \left(\frac{\sqrt{\kappa}}{2}\right)^2 + \left|\frac{\sqrt{\iota}}{2} - \frac{\sqrt{\kappa}}{2}\right|^2,\\ &= \frac{\iota}{4} + \frac{\kappa}{s} + \left|\frac{\sqrt{\iota}}{2} - \frac{\sqrt{\kappa}}{2}\right|^2.\\ \sigma_b(\nu\iota, \nu\kappa) &= \sigma_b\left(\frac{\iota e^{\iota}}{4}, \frac{\kappa e^{\kappa}}{4}\right),\\ &= \left(\frac{\iota e^{\iota}}{4}\right)^2 + \left(\frac{\kappa e^{\kappa}}{4}\right)^2 + \left|\frac{\iota e^{\iota}}{4} - \frac{\kappa e^{\kappa}}{4}\right|^2,\\ &= \frac{\iota^2 e^{2\iota}}{16} + \frac{\kappa^2 e^{2\kappa}}{16} + \left|\frac{\iota e^{\iota}}{4} - \frac{\kappa e^{\kappa}}{4}\right|^2 = B.\end{aligned}$$

$$\begin{aligned}
\sigma_b(\nu\iota, \mu\kappa) &= \sigma_b\left(\frac{\iota e^{\iota}}{4}, \frac{\sqrt{y}}{2}\right), \\
&= \left(\frac{\iota e^{\iota}}{4}\right)^2 + \left(\frac{\sqrt{\kappa}}{2}\right)^2 + \left|\frac{\iota e^{\iota}}{4} - \frac{\sqrt{\kappa}}{2}\right|^2, \\
&= \frac{\iota^2 e^{2\iota}}{16} + \frac{\kappa}{4} + \left|\frac{\iota e^{\iota}}{4} - \frac{\sqrt{\kappa}}{2}\right|^2.
\end{aligned}$$

$$\begin{aligned}
\sigma_b(\nu\kappa, \mu\iota) &= \sigma_b\left(\frac{\kappa e^{\kappa}}{4}, \frac{\sqrt{\iota}}{2}\right), \\
&= \left(\frac{\kappa e^{\kappa}}{4}\right)^2 + \left(\frac{\sqrt{\iota}}{2}\right)^2 + \left|\frac{\kappa e^{\kappa}}{4} - \frac{\sqrt{\iota}}{2}\right|^2, \\
&= \frac{\kappa^2 e^{2\kappa}}{16} + \frac{\iota}{4} + \left|\frac{\kappa e^{\kappa}}{4} - \frac{\sqrt{\iota}}{2}\right|^2.
\end{aligned}$$

Now, by appllying the above inequalities in (25), such that

$$\begin{aligned}
A &= \frac{\iota}{4} + \frac{\kappa}{4} + \left|\frac{\sqrt{\iota}}{2} - \frac{\sqrt{\kappa}}{2}\right|^2, \\
B &= \frac{\iota^2 e^{2\iota}}{16} + \frac{\kappa^2 e^{2\kappa}}{16} + \left|\frac{\iota e^{\iota}}{4} - \frac{\kappa e^{\kappa}}{4}\right|^2, \\
C &= \frac{\iota^2 e^{2\iota}}{16} + \frac{\kappa}{4} + \left|\frac{\iota e^{\iota}}{4} - \frac{\sqrt{\kappa}}{2}\right|^2, \\
D &= \frac{\kappa^2 e^{2\kappa}}{16} + \frac{\iota}{4} + \left|\frac{\kappa e^{\kappa}}{4} - \frac{\sqrt{\iota}}{2}\right|^2,
\end{aligned}$$

we obtains

$$\begin{aligned}
\Gamma + \gamma(F(s^q A)) &\leq F\Big(\eta B + \zeta\frac{C}{2s} + \xi\frac{D}{2s}\Big), \\
\frac{1}{s} + \gamma(F(s^q A)) &\leq F\Big(\eta B + \zeta\frac{C}{2s} + \xi\frac{D}{2s}\Big), \\
\frac{1}{2} + \gamma(F(2^2 A)) &\leq F\Big(\frac{1}{4} \times B + \frac{1}{6} \times \frac{C}{2 \times 2} + \frac{1}{3} \times \frac{D}{2 \times 2}\Big), \\
\frac{1}{2} + \gamma(F(4A)) &\leq F\Big(\frac{B}{4} + \frac{C}{24} + \frac{D}{12}\Big), \\
\frac{4Ae^{\frac{1}{2}}}{1 + 40 \times 4A} &\leq \frac{B}{4} + \frac{C}{24} + \frac{D}{12}, \\
\frac{4Ae^{\frac{1}{2}}}{1 + 160A} &\leq \frac{6B + C + 2D}{24}.
\end{aligned}$$

Hence the inequality (25) satisfied. Thus the mappings μ and ν have a unique common fixed point in Υ, which is $\iota = 0$. Therefore all the conditions of Theorem 3.2 are satisfied.

4. An application of Integral equation using b-metric-like space

In this section, we prove the uniqueness of the solution of the integral equation as an application of Theorem 3.1. Consider the following integral equation inspired by Alghamdi et al. [5] and Chen et al. [12].

$$\iota(t) = h(t) + \int_0^T G(t,s)f(t,s,\iota(s))ds, \forall\, t,s \in [0,T]. \tag{27}$$

where $T > 0$, for all $t,s \in [0,T]$, $f : [0,T] \times [0,T] \times \mathbb{R} \to \mathbb{R}$ and $G : [0,T] \times [0,T] \to \mathbb{R}$ are continuous functions.

Let $\Upsilon = C([0,T],\mathbb{R})$ be the space of all continuous functions defined on $[0,T]$. Notice that $(C([0,T])$ endowed with b-metric-like.

$$\sigma_b(\iota,\kappa) = \sup_{t\in[0,T]} (|\iota(t)| + |\kappa(t)|)^2, \tag{28}$$

for all $\iota,\kappa \in \Upsilon$. Note that (Υ,σ_b) is a complete b-metriclike space with a parameter $s \geq 1$, that is $s = 2$.

We define a mapping $\mu : \Upsilon \to \Upsilon$ by

$$\mu\iota(t) = h(t) + \int_0^T G(t,s)f(t,s,\iota(s))ds,\ t \in [0,T]. \tag{29}$$

If $z \in [0,T]$ is a fixed point of T, then $z \in [0,T]$ is a solution of (27).

Now we prove the following results.

Theorem 4.1. Let $\mu : C([0,T]) \longrightarrow C([0,T])$ be a self maps of a complete b-metric-like space (Υ,σ_b) such that the following conditions hold:

(i) $f : [0,T] \times [0,T] \times \mathbb{R} \to \mathbb{R}$ is a continuous function,

(ii) for all $t,s \in [0,T]$, there exists a continuous function $\mathbb{M}(\iota,\kappa) : [a,b] \times \Upsilon \times \Upsilon \longrightarrow \Upsilon$ with constants $e^{-\tau}$ such that

$$|f(t,s,\iota(s))| + |f(t,s,\kappa(s))| \ \leq\ e^{-\Gamma}\mathbb{M}(\iota,\kappa),$$

where

$$\mathbb{M}(\iota,\kappa) = \frac{(\iota+\mu\iota)^2 + (\kappa+\mu\kappa)^2}{2} = \frac{\sigma_b(\iota,\mu\iota) + \sigma_b(\kappa,\mu\kappa)}{2},$$

(iii) for

$$\left(\sup_{t\in[0,T]} \int_0^T G(t,s)ds\right)^2 \ \leq\ \frac{1}{\gamma 2^q}.$$

Then, the integral equation (29) has a solution.

Proof. By condition $(i), (ii)$ and (iii) of Theorem 4.1, we have

$$\begin{aligned}
|\mu\iota(\mathrm{t}) + \mu\kappa(\mathrm{t})|^2 &= \left(\left|\int_0^T G(t,s)f(t,s,\iota(s))ds\right| + \left|\int_0^T G(t,s)f(t,s,\kappa(s))ds\right|\right)^2, \\
&\leq \left(\int_0^T G(t,s)|f(t,s,\iota(s))|ds + \int_0^T G(t,s)|f(t,s,\kappa(s))|ds\right)^2,
\end{aligned}$$

$$
\begin{aligned}
&\leq \left(\int_0^T G(t,s)(|f(t,s,\iota(s))|+|f(t,s,\kappa(s))|)ds\right)^2, \\
&\leq \left(\int_0^T G(t,s)\left(\frac{(\iota+\mu\iota)^2+(\kappa+\mu\kappa)^2}{2}\right)^{\frac{1}{2}} ds\right)^2, \\
&\leq \left(\int_0^T G(t,s)\left(e^{-\Gamma}\left(\frac{(\iota+\mu\iota)^2+(\kappa+\mu\kappa)^2}{2}\right)\right)^{\frac{1}{2}} ds\right)^2, \\
&= \left(\int_0^T G(t,s)\left(e^{-\Gamma}\left(\frac{(|\iota|+|\mu\iota|)^2+(|\kappa|+|\mu\kappa|)^2}{2}\right)\right)^{\frac{1}{2}} ds\right)^2, \\
&\leq e^{-\Gamma}\mathbb{M}(\iota,\kappa)\left(\int_0^T G(t,s)ds\right)^2, \\
&\leq e^{-\Gamma}\mathbb{M}(\iota,\kappa) \times \frac{1}{\gamma 2^q}, \\
&\leq \frac{e^{-\Gamma}}{\gamma 2^q}\mathbb{M}(\iota,\kappa),
\end{aligned}
$$

$$\sigma_b(\mu\iota,\mu\kappa) \leq \frac{e^{-\Gamma}}{\gamma 2^q}\mathbb{M}(\iota,\kappa). \tag{30}$$

Passing logarithm both sides in (30) and by the property of γ, we get

$$\Gamma + F(\gamma(s^q(\mu\iota,\mu\kappa))) \leq F(\mathbb{M}(\iota,\kappa)). \tag{31}$$

$$\Gamma + F(\gamma(s^q(\mu\iota,\mu\kappa))) \leq F(\frac{\sigma_b(\iota,\mu\iota)+\sigma_b(\kappa,\mu\kappa)}{2}). \tag{32}$$

□

Therefore, the condition imposed in Theorem 4.1 is satisfied for all $z, w \in X$. Hence z is a fixed point of S, also a solution of the integral equation. Thus Theorem 3.1 validated.

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