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COVERAGE PROBLEM IN TWO AND THREE DIMENSION

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Abstract. Coverage in wireless sensor networks (WSNs) is a well known problem. Here we consider that problem in continuous domain. In this paper, we discuss coverage criteria and placement of sensors optimally in $\mathbb{R}^2$ and $\mathbb{R}^3$. Coverage is important in WSNs. WSNs may be two and three dimensional in real life. In practice, sensors usually dropped randomly from space on previously determined positions (called, vertices) of the ROI (Region of Interest). But sensors will not place on the proper vertices in many times. Hence ROI will not be totally covered by the deployed sensors. The question is, how we reduced the area which is not covered by sensors? Usually extra sensors are dropped on some randomly but previously chosen points to minimize the uncovered area. In the current paper, we develop another strategy for deployment of those sensors. We partition the ROI in regular hexagons in two dimension and face-centered cubes in three dimension. Our new strategy is to reduce the side of hexagons or cubes. The amount of reduction depends on the number of extra sensors used. Here we target to deploy exactly one sensor randomly on each vertices. We compare uncovered volume for the two strategies, for two distributions (uniform and normal), and several number of excess sensors used. Simulation result shows that our new strategy is better for lower variance of the randomness but old one is better for higher variance.

1. Introduction

WSNs usually contain a huge number of small sensors (also known as nodes), with some wireless receiver and processing circuit. Usually the the sensors are small. They have minimum battery capacity and processing power. Each sensor contains a low power radio. The sensor measures direction, humidity, speed, distance, temperature, etc. The most important feature of a WSNs is, they can be dropped randomly in an inaccessible region [11]. They also give opportunities for the military and civilian applications; like military tactical surveillance, industrial automation, emergency health care, security of nation, etc. [22]. Sensors are now used in IoT Based Smart Physiotherapy System also [21]. Abdallah et.al. [1] described deployment of sensors for wireless connected things in indoor. Sensors are also used in (monopole) antenna with UHF Band which are hexagonal CSRR grid [16].

Aim of WSNs is monitoring their nearby region for object tracking and event detection. For this reason, *coverage* is important for any wireless sensor network. To fulfil this, a WSNs should cover the Region of Interest, also know as ROI, without any sensing hole [8]. A sensor can detect an event inside a circular region (known as sensing disc) of a prefixed radius (known as sensing radius). A vertex or point will not be covered by a WSN if that point is outside the

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sensing radius of all sensors of the WSN. Our object is to deploy and properly place the sensors in the region, such that the sensors cover whole ROI with least number of wireless sensors or WSN cover more area by a prefixed number of wireless sensors.

However, we cannot expect that the sensors will be placed in a pre-defined manner, as nodes are usually randomly dropped from the air. Wrong placement may happen due to some operational factors. Sensors are deployed in a bounded subset of $\mathbb{R}^2$ or $\mathbb{R}^3$ densely. Also a sensor could damage at any time for several reasons like hardware defects, power depletion, etc. Hence, after ROI is completely covered with the WSN, fault information can be sent by few sensor or they may unable to detect an event. This is known as fault detection. Fault detection may occur due to obstructions or noise. Here we consider only the problem related to coverage of ROI by a set of sensors but not the fault detection problem.

Coverage a ROI problem can be classified into 2 different cases as follows:

- Case 1: ROI (the target region) is partitioned into discrete points i.e., ROI is a grid. The grid may be rectangular with a square as an unit; or it may be a hexagonal (regular) with a regular hexagon as an unit. In this case we want to cover all vertices of the hexagonal grid.
- Case 2: ROI (the target region) is a continuous bounded subset of $\mathbb{R}^2$. In this case we should cover all points of that subset.

In case 1, event can occur at finite number of points only, and in case 2, event can occur at infinite number of points.

There are 2 different ways for deployment or dropping of sensors: (1) placement in a deterministic way and (2) deployment or dropping from the air on a target point. In the first type of placement, ROI may be covered totally by an enough number of nodes or sensors. On the other hand, many points of ROI will not be covered even if there are huge number of sensor randomly deployed. When the sensors is placed in a deterministic way, in a continuous domain, coverage problem is a geometrical problem, and in grid structure, coverage problem is a graph theoretic problem [4].

In case of deployment from air, in general we used robots (actuators) to cover the ROI or to reduce the uncovered area. The above type of network is usually referred as wireless sensor and actuator network (WSAN). In this network, sensors are deterministically placed and relocated by actuators. In some situation, there are movable sensors, and the sensors can put themselves without the help of actuators. But movement of sensors need a huge amount of battery or other energy source, so movement assisted sensor placement is preferred [7, 15]. Some sensors may be find as redundant for coverage of the region, that is, without those sensors the region is covered. Those sensors are known as *passive sensors* and they can deploy deterministically by area coverage protocol [10]. On the other hand, uncovered part of ROI may be covered using the passive sensors, activating them properly.

Coverage is our prime target, but due to lack of nodes, or due to random dropping, or due to fault sensors, we can not avoid uncovered points in a region. On the other hand, actuator may not be available or actuator can not be used in some ROI. In that situation we have to calculate the amount of uncovered area. In this paper, the goal is, develop and calculate some strategies to minimize the uncovered region of ROI.

1.1. **Related Work.** There are several algorithms in literature to place sensors efficiently for covering a convex region in $\mathbb{R}^2$. If the ROI is a bounded convex subset, the problem of covering of ROI is known as *coverage problem* or *covering problem*. Many variations of coverage problem is found in [6]. A survey on the above topics can be found in [13]. There is homological criteria also for covering two dimensional ROI. Fletcher and others develop randomized algorithm using one or more actuators to repair the uncovered region [9]. They describe 2 algorithms in case of grid ROI. They simulate the length of path which is traveled by the actuators. Younis et al. [24] gave a survey on the models and strategies that affect the sensor deployment. Sensor networks are also useful in image processing and for data storage [20]. Dahiya ey.al. develop [3] mobile sink based grid and coverage aware node deployment.

Li et al. discuss the sensor relocation using actuator for coverage [14]. They consider the ROI as grid and use 'virtual force algorithm'. Analysis of maximum and expected distance covered by the actuators to achieve the full coverage can be found in [12, 18]. Deployment of sensors is considered for lattice based three dimensional ROI in [2]. In all the previous literature, the uncovered region is covered, either by dropping extra sensors or using one or more robots, or by activating a group of passive sensors. Nandi et al. develop an new algorithm for robot to minimize the uncovered region [18]. In some networks sensors can detect the position of adjacent nodes and the desired events. The above network is known as a direction sensor network (DSN).

2. Motivations

It is clear that one can classify the covering problem into two broad categories as follows; (i) grid coverage and (ii) coverage in a continuous region. Each categories may be further sub-classified into 4 categories, look at the Table 1:

(1) Deterministic placement of nodes or sensors, and without the help of actuator.
(2) Random deployment of nodes or sensors, and use of one or more actuators.
(3) Random deployment of nodes or sensors, and use of some extra sensors.
(4) Random deployment of nodes or sensors along with use of some extra sensors and one or more actuators.

Table 1. Classification of covering problem and previous work

Modes of deployment	Type of the ROI	
	Grid structures	Continuous region
Deterministic	[5]	[17], In current paper
Random deployment and actuators	[2, 9, 13, 24]	[9, 13, 14, 24]
Random deployment and extra sensors	[12, 18]	[17], In current paper
Random deployment and extra sensors and actuators	[12, 18]	Still open

Many research was done on the following problem: 'Whether the ROI is totally covered or not?'. Moreover, if the region is not fully covered by nodes then there are methods to cover the region using robots, extra sensors, movable sensors ect. But there is no work till date on the following problems: (i) 'How the uncovered region changed with respect to the number of nodes?' and (ii)'How the uncovered region depends on the strategy of random dropping of the nodes?'. If the number of nodes is not enough to cover ROI, or number of nodes is sufficient to cover the full region in the deterministic way, in that situation also, there may be some uncovered region. We cannot give guarantee on the full coverage of the region. Moreover, if we drop some extra sensors total covering is not guaranteed unless we relocate of the nodes either by movable sensors or by actuator(s). In this paper, we consider that there is no mobile sensor or actuator. Hence our main target is to minimized or reduce the uncovered area.

Now it is sufficient to cover each vertix or point of the ROI by not more than one node. Hence if some portion of the region is covered by greater than or equal to two nodes then it is in some sense'wastage'. But since the sensing area of a node is a disc, hence we can not have zero wastage. So our goal is to reduce the wastage portion of the region. One general idea is, deploy the nodes in some deterministic points of ROI, such that if they are really placed on that pre-fixed points, in that case, the wastage is minimum. But after deployment there will be uncovered area due to the random deployment of the nodes. So we require some extra sensors.

Now the problem can be stated as follows, how we drop the sensors such that the wastage is minimum i.e., how we use the extra nodes to reduce the uncovered area. Nandi and Sarkar find the solution of the above problem in $\mathbb{R}^2$ [17]. In this paper we briefly recall all the results of that paper and find solution in $\mathbb{R}^3$

2.1. **Our Contributions.** In this paper, we have considered two and three dimensional coverage problem in continuous region. Now we state the second problem formally as follows:

For an arbitary index set J, consider the set of unit balls $\{C_j \subseteq \mathbb{R}^n : j \in J\}$, which cover a convex and bounded subset of $\mathbb{R}^n$. This set is considered to be ROI. Consider a collection of n-dimensional random vectors $\{Y_j : j \in J\}$. Let D_j is the distance between Y_j and the center of C_j, for $j \in J$. Assume that D_js are i.i.d. with p.d.f. $f(\cdot)$. Now the question is 'what portion of ROI will be uncovered?' Here we consider two probability distributions for D_i, normal and uniform. We consider 2 different strategies for deployment for the extra nodes or sensors. We consider face-centered cube as it is optimal for coverage problem in $\mathbb{R}^3$.

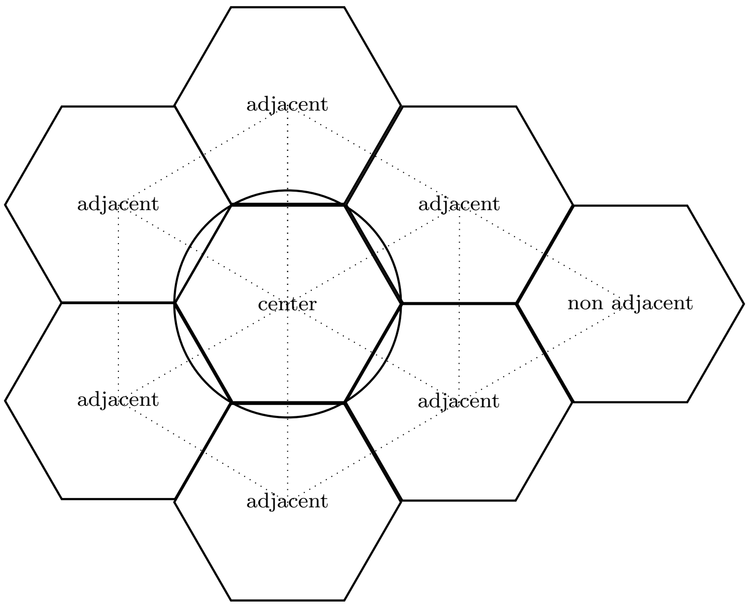


FIGURE 1. Nodes placed in hexagonal grid when ROI partitioned into regular hexagons

In the current paper we consider two strategies for dropping or deployment of the extra sensors. One strategy (call it St.1) is that, deploy exactly one sensor at the target points. Next choose some points deterministically or randomly (depending on the number of extra sensors) and drop or placed one extra sensor on those points. The second strategy (say St2 or St. 2) is as follows: reduce the distance between two adjacent points (depending on the number of extra sensors) and drop or place exactly one sensor on every target points. We calculate using simulate and numerically (when there are no extra sensor) the covered area (in 2-dimension) or volume (in 3-dimension). We observed from result that, St.1 is better when the variance of probabilty distribution of D_j is large and St.2 is better when the variance of probabilty distribution of D_j is small.

Our assumption is, a node can be dropped at an arbitrary point of ROI. We also assume, the distance between the point where the node placed and the target point where we want to place, is a random variable. Now when we drop extra sensors at some randomly or deterministically chosen point then the proportion of the uncovered region (area) will decrease. On the other hand, if we decrease the distance between the neighboring target points, but keeping the sensing radius unchanged, and placed exactly one sensor at each point, then also the uncovered area will decrease. The idea is, we use the extra sensors in 2 different ways in 2 different strategies. Two basic differences between the 2 strategies are as follows:

- In the first strategy, (say, St.1), we target to deploy two sensors on few randomly chosen centers and one sensor on to the rest. In the second one, (say, St.2), we deploy exactly one node or sensor on the target points.
- Let in St.1, there are n hexagons and k extra sensors are used, that is, total $n+k$ sensors is used in St.1. If the length of the sides is a in St.1, then in St.2 the length of the side will be b such that $(n+k)b^2 = na^2$. Hence the total area target to cover is same in the both cases. In three dimension the relation will be $(n+k)b^3 = na^3$. Hence the distance between two target vertices is less in St.2.

Note that, the sensing radius and number of sensors are equal for both the strategies.

In this paper we consider ROI as convex bounded subsets of $\mathbb{R}^n$, with extra emphasis on $n = 2, 3$. The distance between the target point for a node and the point where the node is placed after deployment, considered as a random variable (D_i) whose probability distribution is either normal or uniform. We simulate and calculate the proportion of the uncovered area for above two distributions and for both strategies. We compare thees strategies with respect to the uncovered area.

In coverage problem, usually hexagonal or square partition of the region is used. It is known that partitioning the ROI into regural hexagons is better than the other. But after random deployment of extra nodes, hexagonal partition may not be better than the strategy of partitioning the ROI into congruence squares. In this paper, we consider the hexagonal partition only. Similar words can be said for cube centered partition in 3 dimension case. But we consider the cube centered partition only for 3 dimension.

We calculate using simulation, the proportion of uncovered volume (in 3 dimension case) and area (in 3 dimension case) for 2 distributions (normal and uniform) and two aforesaid strategies for deployment of extra nodes.

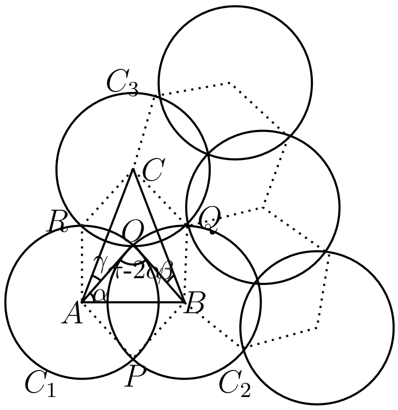


Figure 2. Hexagonal tiling of ROI

3. Assumptions and Definitions

Assume the ROI is partitioned into a number of congruent cube in 3 dimension and into a number of congruent regular hexagons of side length a in 2 dimension. To cover the each hexagon using exactly one node or sensor we must take $a \leq r$ (r be the sensing radius). If $r = a$ each hexagon will be covered by exactly one sensor when it is placed on the center of the hexagon. Assume that the sensors are too small and that can be think as a point. Next we define few important terms.

- *Node* is that point where a particular sensor is placed after the deployment. We use the word 'node' to mean that point where a typical sensor placed, as well as the respective sensor.
- *Vertex* is that point where a sensor is to target to place.
- $N(W)$ is the node which corresponds to a vertex W, that is, a sensor is placed on $N(V)$ but the target was to place at W.
- $V(M)$ is the respective vertex of a node M.
- *Sensing Disc* S_N of a node N is a closed disc of radius r with center N, which is covered by the sensor placed at that node.
- The radius of the disc, r is known as *sensing radius*. Sensing radius is assumed to be same for all discs. Throughout the paper, by the word 'disc' we consider closed discs only. In higher dimensions we call it as *sensing ball*.
- *Adjacent vertex* of a vertex is that vertex which is at the distance not more than twice of the sensimg radius from the aforesaid vertex. Therefore the sensing disc of a node has non empty intersection with the sensing disc of its adjacent nodes and empty intersection with the sensing disc of a node which is not an adjacent node.
- $\mathcal{W}$ is set of all vertices and Adj_W is set of all the adjacent vertices of vertex W (see Figure 1). Similar definitions and notations apply for nodes also. The respective notations are $\mathcal{N}$ and Adj_N for $N \in \mathcal{N}$.
- The distance between two points A and B is denoted by $d(A, B)$.
- A point $A \in \mathbb{R}^n$ is said to be covered by a node N if $d(A, N) \leq r$ and the point A is said to be covered by a set of nodes $\mathcal{N}$ if A is covered by at least one node in $\mathcal{N}$. A point $A \in \mathbb{R}^n$ is said to be uncovered by a node N if it not covered by N and the point P is said to be uncovered by $\mathcal{N}$ if P is not covered by any nodes in $\mathcal{N}$.

- ROI will be called covered by a set of nodes if every point of ROI is covered by at least one node.
- Volume of a set S will be denoted as $\mathrm{Vol}(S)$.

Observe that if there is no randomness, that is, sensor are placed on exact point, then a vertex and its corresponding node is same, $V(N) = N$ and $N(V) = V$.

We shall now define the most important term called 'wastage'. Let S be any bounded set in $\mathbb{R}^n$, which is covered by a finite set of sensors or nodes $\mathcal{N}$. The wastage in S for $\mathcal{N}$ is define as follows

$$W_{\mathcal{N}}(S) = \frac{\displaystyle\sum_{M\in\mathcal{N}} \mathrm{Vol}(S\cap S_M) - \mathrm{Vol}(S)}{\displaystyle\sum_{M\in\mathcal{N}} \mathrm{Vol}(S\cap S_M)}.$$

If $\mathcal{N}$ is a set so that $|S_{N_1}\cap S_{N_2}\cap S_{N_3}| \leq 1$ for distinct $N_1, N_2, N_3 \in \mathcal{N}$, then

$$W_{\mathcal{N}}(S) = \frac{\displaystyle\sum_{N_1\neq N_2\in\mathcal{N}} \mathrm{Vol}(S\cap S_{N_1}\cap S_{N_2})}{\displaystyle\sum_{M\in\mathcal{N}} \mathrm{Vol}(S\cap S_M)}.$$

Intuitively, the denominator of the above expression is the total volume, which is common with S, and the numerator is the wastage in volume. Here 'wastage' represent the ratio of wastage volume and the total volume.

Let $\mathcal{N}$ be the set of sensors or nodes which cover whole $\mathbb{R}^n$ and $\mathcal{N}\cap S$ is a finite set for all bounded set S. We define wastage in $\mathbb{R}^n$ for $\mathcal{N}$ as follows

$$W_{\mathcal{N}}(\mathbb{R}^n) = \lim_{y\to\infty} W_{\mathcal{N}\cap B_y}(B_y),$$

where B_y be the ball in $\mathbb{R}^n$ of radius y and centered at origin.

Intuitively, wastage in $\mathbb{R}^n$ is the proportion of wastage volume in $\mathbb{R}^n$. Note that we can take an increasing sequence of sets whose limit (union) is $\mathbb{R}^n$ other than B_x, e.g., for $n = 2$ partitioned $\mathbb{R}^2$ into hexagons and then take an increasing sequence of union of finitely many such hexagons with the property that limit (union) of this sequence is $\mathbb{R}^2$. In that case we can define wastage similarly. It can be proved that these two definitions are equivalent.

4. Hexagonal Placement of Nodes

In the Hexagonal placement wastage is $1 - \frac{3\sqrt{3}}{2\pi}$, which is closed to 0.163. We know that the area of a regular hexagon with side a is $\frac{3\sqrt{3}}{2}a^2$. So number of nodes required to cover a region of area T which is partitioned into congruent regular hexagons of side a is $m = \frac{2T}{3\sqrt{3}a^2}$, where a is the sensing radius.

In real situation ROI is finite. Let we have M discs with radius a. We want to cover maximum area of ROI with these discs. Assume that the coordinates of the center of the first disc is (0,0). In the following algorithm (Algorithm 1), we describe the placement of the nodes.

Input: M and a (M=Total number of discs, a=radius of a disc).
Input: L, (L=Maximum number of discs can be placed in a row).
Output: Coordinates of the center of the discs.

$T = 2L - 1$;
$Y[M], Z[M], j = 0, r = 0$;
while $j < M$ **do**
    if $j \bmod T = 0$ *and* $j > 0$ **then**
        $r = r - \sqrt{3}a$;
    end
    if $j \bmod T < L$ **then**
        $Y[j] = (j \bmod T)3a$;
        $Z[j] = r$;
    end
    else
        $Y[j] = \Big((j + L - 1) \bmod T \Big) 3a + \frac{3a}{2}$;
        $Z[j] = r - \frac{\sqrt{3}a}{2}$;
    end
    $j = j + 1$;
end
Report y coordinates $Y[M]$ and z coordinates $Z[M]$ of discs;

Algorithm 1: Placement of nodes in Hexagonal grid

5. Coverage Problem in $\mathbb{R}^2$ with Maximum Coverage

Now we state some relevant results finds in [17] without proof.

Theorem 1. *Let a node or sensor in $\mathbb{R}^2$ can detect the position and distance (ith respect to a coordinate system) of adjacent nodes. Assume that adjacent nodes are placed at distance s with $0 < s < 2r$, where r is the sensing radius. In this scenario sensors can detect the near by sensing holes.*

Theorem 2. *Let a sensor be targeted to be placed at B, but it is placed at C on the plane i.e., $C = N(B)$. Assume the distribution of the distance between B and C have the density $g(\cdot)$. Let D be a point at a distance l from B. Then the probability that the point D is in the sensing disc of the node C is*

$$\int_0^{r-l} f(x)dx + \frac{1}{\pi}\int_{r-l}^{r+l} \cos^{-1}\left(\frac{l^2 + x^2 - r^2}{2xl}\right) f(x)dx, \quad \text{if } 0 \le l \le r$$

and

$$\frac{1}{\pi}\int_{l-r}^{l+r} \cos^{-1}\left(\frac{l^2 + x^2 - r^2}{2xl}\right) f(x)dx, \quad \text{if} \quad l > r.$$

Theorem 3. *Let a sensor be targeted to dropp at a point B, but it placed randomly at any point in the disc of radius r and center B. Let that random point be B. Let E be a point at a distance d from B. Then the probability of the event that the point E belong to the sensing disc of the node C is*

$$\frac{2}{\pi}\cos^{-1}\left(\frac{d}{2r}\right) - \frac{d}{2\pi r^2}\sqrt{4r^2 - d^2} \quad \text{if} d \le 2r, \quad \text{and 0 otherwise.}$$

Theorem 4. *Let there are n nodes and the target vertices are the centers B_i, $\forall, i = 1, \ldots, m$; of the m regular hexagons so that i^{th} node is targeted to place on the center B_i. But the i^{th} node is placed at point E_i. Let E_i's be i.i.d. with uniform distribution on the circular disc of radius r and centered at B_i. Then the ratio of the expected covered area is closed to*

$$1 - \frac{8}{\sqrt{3}} \int_{x=0}^{\frac{1}{2}} \int_{y=0}^{\sqrt{3}(\frac{1}{2}-x)} \prod_{i=1}^{7} \left(1 - \frac{2}{\pi}\left(\cos^{-1}(l_i) - l_i\sqrt{1-l_i^2}\right) I_{(0,1)}(l_i)\right) dydx,$$

where l_i is the half of the distance between the points C and Q_i and the coordinates of C and Q_i's are $(x, y), (3/2, 0), (3/2, \sqrt{3}), (0, -\sqrt{3}/2), (0, \sqrt{3}/2), (0, 3\sqrt{3}/2), (-3/2, \sqrt{3}), (-3/2, 0)$, respectively $\forall i = 1, \ldots, 7$.

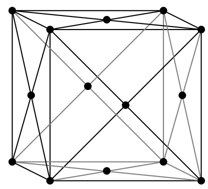


FIGURE 3. Face-Centered Cube where the dots are center of the spheres

6. Coverage Problem in $\mathbb{R}^3$ with Maximum Coverage

It is well known that the face-centered cube packing (see Figure 3) is optimal placement of nodes for sphere packing problem in $\mathbb{R}^3$. In several situations sensor network is 3 dimensional. Here we discuss a similar placement of sensors (as in face centered cube packing) to cover $\mathbb{R}^3$. Consider the set $\mathcal{S} = \{(2l, 2m, 2n) : l, m, n \in \mathbb{Z}\} \cup \{(2l+1, 2m+1, 2n) : l, m, n \in \mathbb{Z}\} \cup \{(2l+1, 2m, 2n+1) : l, m, n \in \mathbb{Z}\} \cup \{(2l, 2m+1, 2n+1) : l, m, n \in \mathbb{Z}\}$.

Partition $\mathbb{R}^3$ in unit cube and put the sensors at the eight corners and the center of the six faces of the cubes. Let the sensing radius be r. Consider the set of sensors $\{rM : M \in \mathcal{S}\}$. The placement of sensors here is same as the choice of center of spheres in the face-centered cube packing. Only difference is that the distance between the 2 adjacent sensors is less in this case to confirm the coverage.

Theorem 5. *Consider a part of cube K of side $2mr$ unit into m^3 cubes of side $2r$ unit. Then number of sensors required to cover the cube K is $4m^3 + 6m^2 + 3m + 1$, where the sensors are placed as discussed above (similar to face-centered cube). The proportion of wastage volume is $1 - \frac{8m^3}{(4m^3+6m^2+3m+1)\times\frac{4}{3}\pi}$, which is approximately $1 - \frac{3}{2\pi}$ for large m.*

Proof. Observe that there are $(m+1)^3$ corner sensors and $m^2(m+1)$ sensors at the center of faces which are parallel to one of the coordinate planes. Hence total number of sensors is $(m+1)^3 + 3m^2(m+1) = 4m^3 + 6m^2 + 3m + 1$.

We need $4m^3 + 6m^2 + 3m + 1$ spheres of radius r to cover the cube of side $2mr$. Hence total volume of all spheres is $(4m^3+6m^2+3m+1)\frac{4}{3}\pi r^3$ and they cover volume of $8m^3r^3$ units. Hence the ratio of wastage volume is $\frac{(4m^3+6m^2+3m+1)\times\frac{4}{3}\pi r^3 - 8m^3r^3}{(4m^3+6m^2+3m+1)\times\frac{4}{3}\pi r^3}$. Which complete the proof. □

7. Simulation Result

In this section, we describe the simulation procedure and the data we get from these simulations for two and three dimension.

7.1. **Simulation for two dimension.** Observed that the radius of the sensing disc (r) has no role in simulation study. We consider, in our simulation, 10000 nodes with $r = 1$. Partition ROI as regular hexagon and try to deploy a sensor at the center of each regular hexagon. Clearly the total area is $10000 \times \frac{3\sqrt{3}}{2}$ unit. Also two adjacent vertices has distance $\sqrt{3}$ unit. 100 sensors are placed in each row. We assume that the distance D_i between the target vertex and respective node are i.i.d. either uniform or normal. Let $p\%$ extra sensors have been used, hence the total number of sensors is $10000(1 + \frac{p}{100})$ where $p \in [0, 100]$.

Next we want to simulate the uncovered proportion. The first strategy (call, St 1) is as follows: pick $100p$ vertices unifomly and randomly from the 10000 and deploy two sensors on to each of the selected vertices and deploy one sensor for other $(10000 - 100p)$ vertices. Then we simulate the ratio of uncovered area of ROI and we repeat the simulation 10000 times and take the average of the ratios.

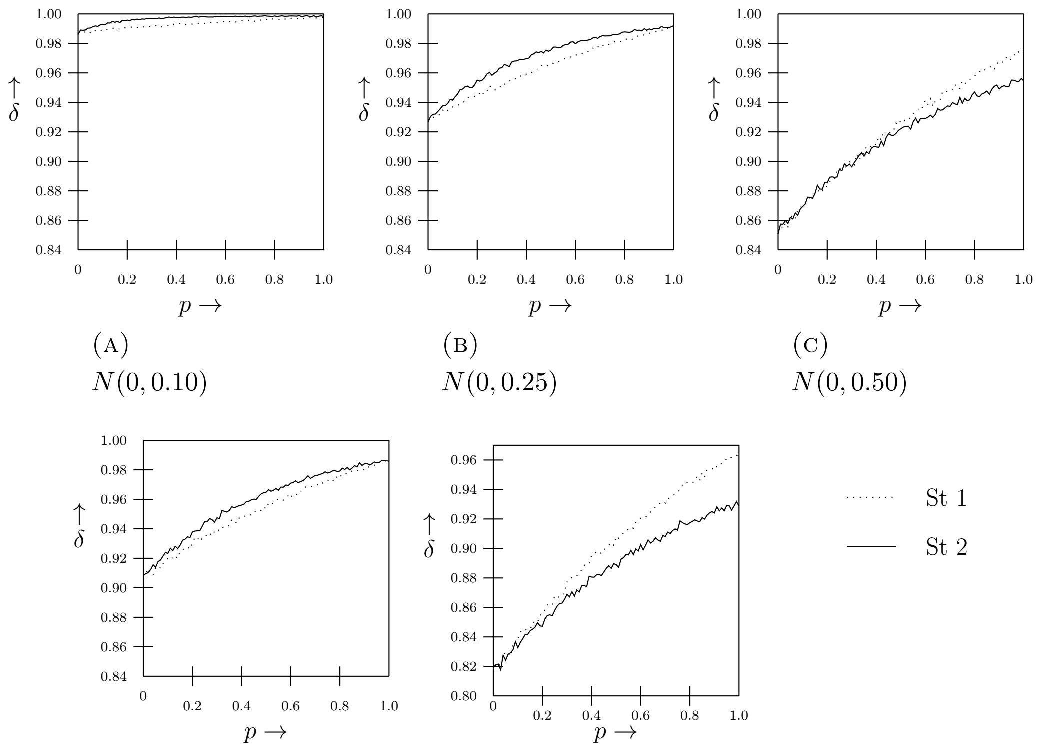
The second strategy (call, St 2) is as follows: partition ROI in to $10000(1 + \frac{p}{100})$ regular hexagon of side $\sqrt{\frac{100}{100+p}}$. Consider $10000(1 + \frac{p}{100})$ centers of these hexagons as vertices and deploy one sensor exactly for every vertices. Observe that the area of the whole region is $10000(1 + \frac{p}{100}) \times \frac{3\sqrt{3}}{2}\left(\sqrt{\frac{100}{100+p}}\right)^2$, which is same as the St. 1. Next we shall simulate the propotion of uncovered area as simulate before.

Let D_i be the distances from the target vertex to the respective node. In simulation, we consider 5 different distributions for D_is. D_is are considered to be independent. The uniform distribution whose p.d.f. is $f(x) = \frac{2x}{t^2} I_{(0,t)}$ denoted by $U(t)$ and $N(0, t^2)$ be the normal distribution with expectation 0 and sandard deviation t. We compare the simulated uncovered area for 9 distinct values of p. We also draw the 'proportion of covered area (δ)' vs. 'p' graphs for five distributions (see Figure 4a to 4e).

From the simulated data it is noted that St 1 is better for $U(1.0)$ and $N(0, 0.50)$ distribution but St 2 is better in case of other 3 distributions. Hence we conclude that St 1 is better for high standard deviation but St 2 is better for low s. d. We also calculate the proportion of coverage area for $U(1)$ and $p = 0$ numerically (see theorem 4). We observed that the value is near to the corresponding value what we get from the simulation.

7.2. **Simulation for three dimension.** We consider $n = 13$ and $r = 1$ (n and r as in theorem 7). Hence there are 9842 nodes. The volume of the ROI is $13^3 \times 2^3$ unit. We want to calculate using simulation the proportion of the volume covered by the sensors for two different strategies, St. 1 and St. 2. Consider that we shall use $p\%$ extra sensor. Hence for St.1 we deploy two sensors on each of $p\%$ target vertices and one sensor on the rest. For Strategy 1 we must partition the ROI with volume $13^3 \times 2^3$ unit into m^3 many cubes where $4m^3 + 6m^2 + 3m + 1 = 9842 \times (1 + \frac{p}{100})$ (see theorem 7) and deploy exactly one sensor on each target vertices. Note that for St.2, we reduced the length of the cube such that number of target nodes increased by p percent.

We calculate, using simulation, the proportion of the coverage volume for 2 strategies, and for 5 different distributions, (as in case of $\mathbb{R}^2$ described in the previous subsection) and for nine different values of p (see Table 2). We know the uniform distribution has density function $f(x) = \frac{3x^2}{t^3} I_{(0,t)}$, where t is the parameter. We observed from the data (from simulation) that, St. 1 is better than St. 2 when the distribution has higher variance. For distribution with low variances, St. 2 is better for almost all values of p. This observation is same as in two



(A) $N(0, 0.10)$ (B) $N(0, 0.25)$ (C) $N(0, 0.50)$

(D) $U(0.5)$ (E) $U(1.0)$

FIGURE 4. Graph (from simulation data) of proportion of coverage area in $\mathbb{R}^2$

dimension. Hence we draw the consion that St. 1 is better in case of higher variance but St. 2 is better in case of lower variance.

TABLE 2. Simulation result of proportion of the coverage volume in $\mathbb{R}^3$

	$U(0.50)$		$U(1.00)$		$N(0, 0.100)$		$N(0, 0.250)$		$N(0, 0.500)$	
p	St 1	St 2	St 1	St 2	St 1	St 2	St 1	St 2	St 1	St 2
0.000	0.97090	0.97091	0.92350	0.92332	0.96579	0.96421	0.93848	0.93468	0.93483	0.93302
0.050	0.96901	0.97103	0.92904	0.92918	0.96849	0.96881	0.94401	0.94550	0.94079	0.93270
0.100	0.97590	0.97371	0.93380	0.93350	0.96951	0.97079	0.94700	0.94750	0.94380	0.94561
0.150	0.97752	0.97811	0.94231	0.93680	0.97230	0.97791	0.95402	0.95048	0.95281	0.94580
0.200	0.98112	0.98251	0.94601	0.94500	0.97968	0.97649	0.96184	0.95502	0.95303	0.95182
0.250	0.98527	0.98502	0.95676	0.94392	0.98230	0.97870	0.96452	0.95770	0.96201	0.96001
0.500	0.99179	0.99150	0.97300	0.96247	0.99132	0.98594	0.97873	0.97052	0.98029	0.96690
0.750	0.99550	0.99450	0.99030	0.97071	0.99579	0.99361	0.99301	0.98171	0.98701	0.97850
1.000	0.99850	0.99751	0.99447	0.97948	0.99802	0.99520	0.99530	0.98649	0.99428	0.98271

8. CONCLUSION

In the current paper, we try to solve the coverage or covering problem in $\mathbb{R}^3$. We have described different coverage criteria and studied the number of sensors require to cover a region. We also consider that sensors or nodes may not be properly placed at the required target point but may be placed at any point in the plane. We assume that the distance between these two points follows i.i.d. We consider the distribution either uniform or normal. For the case of uniform we calculate theoretically the uncovered area of the region. For these two distributions we have done computer simulations. To reduce the uncovered volume we have introduced two different strategies using extra sensors and have compared these two strategies. We see that the first strategy is better for distributions whose have higher variance and the second strategy is better for distributions whose have smaller variance. We notice that there are two different aspects of the coverage problem:

(1) Placement of nodes is random in real life situations and one can model or fit the data in to a probability distribution.
(2) We develop two methods for dropping for extra nodes. There are other optimal strategies depending on different distributions, different type of ROI and different methods for placement of sensors.

Now we list some possible future works:

- In this paper we used two types of distributions; Uniform and Normal. We find some theoretical results for uniform distribution but no result is found till date for Normal distribution. Some theoretical formula may be develop in case of normal p.d.f.
- In this paper we consider two and three dimensions. In future one may consider the optimal placement of sensors for higher dimensional coverage problem.
- In future one may think ROI as a square grid structure and the distributions like two dimensional exponential distribution for deployment.
- In this paper we consider 2 strategies. There are many others, which may be better, for other distributions. In future one can classify the distributions and strategies w.r.t. different types of partitions and distributions.
- In this paper we did not use the actuators. In future one may try to solve coverage problem using extra sensors and actuators.

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